Section 6.4   Metric System (Lengths)

The metric system is part of the international system of units (SI system) that defines units to measure lengths, volumes, masses, times, amounts, electric currents, and luminous intensities. The meter is the basic unit for the measurement of lengths in the metric system. A meter was originally defined in 1791 as one ten-millionth of the estimated length of the meridian line from the North Pole to the equator that runs through Paris, which is roughly ¼ the circumference of the earth. In 1983, the meter was redefined more precisely as the length of the path traveled by light in a vacuum during one-299,792,458[th] of a second. To get a feel for the length of a meter think of a meter as a little larger than a yard, a little larger than three and a quarter feet, or a little less than 40 inches. Below, a meter is defined in terms of U.S. customary units.

$$1 \text{ meter} \approx 1.09 \text{ yards} \approx 3.28 \text{ feet} \approx 39.4 \text{ inches}$$

What makes the metric system so appealing is that it is a decimal system. Thus, converting lengths within the metric system is a simple calculation performed by multiplying or dividing by a place value number such as 10, 100, or 1000 which is a very easy process as compared to the complicated U.S. customary system of units for lengths where 12 inches equals one foot, 3 feet equals one yard, and 1760 yards equal one mile. Below are some of the basic metric prefixes with the most commonly used metric prefixes bolded.

kilo	hecto	deca
Thousand times larger	Hundred times larger	Ten times larger

deci	**centi**	**milli**
Ten times smaller	**Hundred times smaller**	**Thousand times smaller**

In the table below one dollar is inserted as the standard unit to serve as a visualization tool for metric prefixes. Notice the resulting value that the metric prefix dollars are assigned in terms of the U.S. currency system.

Kilo-dollar	Hecto-dollar	Deca-dollar	dollar	Deci-dollar	**Centi-dollar**	**Milli-dollar**
$1000	$100	$10	$1	$.10 (dime)	**$.01 (1 cent)**	**$.001 (0.1 cent)**

1 **kilo**-dollar = **1000** dollars

1 dollar = **10 deci**-dollars = **100 centi**-dolars = **1000 milli**-dollars

The table below shows the basic metric units for measuring lengths as well as lists their abbreviations. Notice the decimal nature of the metric system with 10 of each unit being equal to 1 of the adjacent larger unit. Memorize the bolded entries.

kilometer	hectometer	decameter	meter	decimeter	**centimeter**	**millimeter**
km	hm	dam	m	dm	**cm**	**mm**
1000 meters	100 meters	10 meters	1 meter	0.1 meters	**0.01 meters**	**0.001 meters**

1 kilometer is equivalent to 1000 meters   (approx. 0.6 miles)

1 hectometer is equivalent to 100 meters

1 decameter is equivalent to 10 meters

1 meter ≈ 1.09 yards ≈ 3.28 feet ≈ 39.4 inches

1 decimeter is one-tenth of a meter, so 10 decimeters equals 1 meter

────────────────────────────────────── (1 dm)

1 centimeter is one-hundredth of a meter, so **100 centimeters equals 1 meter**

▬▬ (1 cm)

1 millimeter is one-thousandth of a meter, so **1000 millimeters equal 1 meter**

▪ (1 mm)

To get a feel for some of the basic metric lengths, one millimeter is about the width of a dot, one centimeter is about the width of a pinky finger, one meter is a little over a yard in length, and a kilometer is more than half a mile. Notice that for millimeters, 10 millimeters (dots) equal a centimeter (width of a pinky finger) and 1000 millimeters (dots) equal the length of a meter. As for centimeters, 100 centimeters (width of a pinky finger) equal the length of a meter.

1 meter = 10 decimeters = **100 centimeters** = **1000 millimeters**

Example 1   Insert the appropriate metric length unit.

A scenic hiking trail is 8 _____ long
Since a hiking trail is usually measured in miles the unit is kilometers.

An ant is about 3 _____ long
A typical ant is pretty small about 3 dots in length so the unit is millimeters.

An average basketball player has a height of 2 _____
A basketball player is usually over 6 feet tall, so the unit is meters.

The length of a standard pencil is 19 _____
The length of a pencil is about the width of 19 fingers, so the unit is centimeters.

Below conversion factors are listed that connect the metric length units of kilometer, meter, and centimeter to U.S. customary length units of miles, yards, feet and inches.

1 kilometer $\approx$ 0.62 miles	1 meter $\approx$ 1.09 yards	1 inch = 2.54 centimeters
1 mile $\approx$ 1.6 kilometers	1 meter $\approx$ 3.28 feet	
	1 meter $\approx$ 39.4 inches	

The first type of conversion problems is going from U.S. customary length units to metric units and vice versa using given conversion factors. To convert using the dimensional analysis technique the calculation is written as a product of fractions with the conversion factor rate written as a fraction with the output unit (converted to unit) in the numerator and input unit (converted from unit) in the denominator. When the multiplication calculation shown below is performed the converted from unit cancels out leaving the appropriate converted to unit as the final answer. The follow phrase describes this process, "write the conversion factor rate with the converted to unit on the top".

$$\overbrace{\left(\frac{converted\ to\ unit}{\cancel{converted\ from\ unit}} \right)}^{conversion\ factor} \times \frac{\left(\cancel{converted\ from\ unit} \right)}{1}$$

Example 2   Convert 8 feet into meters

The convert to unit is meter.

Conversion factor written with the converted to unit meter on the top $\dfrac{1 \text{ km}}{0.62 \text{ mi}}$

This rate multiplied times the 8 feet results with the foot unit being canceled as shown below.   Write the resulting fraction 8/3.28 in decimal form.   8 feet is approximately 2.44 meters.

$$\frac{1 \text{ m}}{3.28 \text{ ft}} \times \frac{8 \text{ ft}}{1} \;=\; \frac{8}{3.28} \text{ m} \;\approx\; 2.44 \text{ m}$$

Example 3   Convert 3 inches into centimeters

Convert to unit is centimeters and covert from unit is inch.

Conversion factor written with the converted to unit centimeter on the top $\dfrac{2.54 \text{ cm}}{1 \text{ in}}$

This rate multiplied times the 3 inches results with the inch unit being canceled as shown below. To evaluate this expression multiply the numerators 2.54 and 3 whose product 7.62 centimeters is approximately 3 inches.

$$\frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{3.00 \text{ in}}{1} \;\approx\; 7.62 \text{ cm}$$

Example 4   The largest recorded python snake is 6.95 meters in length. Find the length of this python snake in feet.

The convert to unit is feet.

Conversion factor written with the converted to unit meter on the top $\dfrac{3.28 \text{ ft}}{1 \text{ m}}$

This rate multiplied times the 6.95 meters results with the meter unit being canceled as shown below. To evaluate this expression multiply the numerators 3.28 and 6.95 whose product 22.8 feet is the length of the largest recorded python snake.

$$\frac{3.28 \text{ ft}}{1 \text{ m}} \times \frac{6.95 \text{ m}}{1} \;\approx\; 22.8 \text{ feet}$$

Example 5   The length of a marathon race is 26.2 miles. Find the length of a
               marathon race in kilometers.

The convert to unit is kilometer.

Conversion factor written with the converted to unit kilometer on the top $\dfrac{3.28 \text{ ft}}{1 \text{ m}}$

This rate multiplied times the 26.2 kilometers results with the kilometer unit being
canceled as shown below. Write the resulting fraction 26.2/0.62 in decimal form. A
marathon is approximately 42.2 kilometers long.

$$\frac{1 \text{ km}}{0.62 \text{ mi}} \times \frac{26.2 \text{ mi}}{1} = \frac{26.2}{0.62} \text{ km} \approx 42.2 \text{ km}$$

For some conversions a direct conversion factor between the converted to unit and
the converted from unit may not be listed in a reference table, but other conversion
factor are listed which serve as a middleman between these units. In these cases
multiple conversion factors are employed and arranged in such a way that all units
besides the output unit cancel out.

Example 6   Convert 40 centimeters into feet using dimensional analysis.

A direct conversion factor between centimeters and feet is not listed in this section.
Thus, the inch unit will serve as the middleman connecting the feet and centimeter
unit. First convert the centimeters into inches then convert the inches into feet.
Write the conversion factors 1 inch equals 2.54 centimeters and 1 foot equals 12
inches in fraction form to create the expression shown below with the convert to unit
feet in the numerator. To evaluate find the product of the denominators 12 and 2.54
which equals 30.5 then write the resulting fraction 40/30.54 in decimal form. 40
centimeters is approximately 1.3 feet.

$$\frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{91.5 \text{ cm}}{1} = \frac{40}{30.54} \text{ ft} \approx 1.3 \text{ ft}$$

The efficiency of the metric system becomes evident when converting units within the metric system. The resulting dimensional analysis calculation for these metric conversions involves multiplication or division by a place value number which can be written in the form $(10)^n$ which is easily performed without a calculator by simply moving the decimal point n places either to the right or left.

Multiplication by 10 means moving the decimal point 1 place to the right.

Multiplication by 100 means moving the decimal point 2 places to the right.

Multiplication by 1000 means moving the decimal point 3 places to the right.

Division by 10 means moving the decimal point 1 place to the left.

Division by 100 means moving the decimal point 2 places to the left.

Division by 1000 means moving the decimal point 3 places to the left.

Example 7   Do the following calculations mentally in your head.

1.25 × 1000   2.7 ÷ 100   14.3 × 10   24.5 ÷ 1000

1.25 × 1000 = 1,250
Move the decimal place three places to the right.

2.7 ÷ 100 = 0.027
Move the decimal place two places to the left.

14.3 × 10 = 143
Move the decimal place two places to the right.

24.5 ÷ 1000 = 0.0245
Move the decimal place three places to the left.

Below the conversion factors are listed for the most commonly used metric length units. All these conversion factors involve place value numbers which result in multiplications and divisions which can be performed without a calculator by simply moving the decimal point the appropriate number of places to either the right or left.

1 kilometer = 1000 meters	1 meter = 100 centimeters	1 centimeter = 10 millimeters
	1 meter = 1000 millimeters	

Example 8   Convert 1.32 meters into centimeters

The convert to unit is centimeter.

Conversion factor written with the converted to unit centimeter on the top $\dfrac{100 \text{ cm}}{1 \text{ m}}$

This rate multiplied times the 1.32 meters results with the meter unit being canceled as shown below. To multiply 1.32 by 100 simply move the decimal point two places to the right. 1.32 meters equals 132 centimeters

$$\frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1.32 \text{ m}}{1} = 132 \text{ cm}$$

Example 9   Convert 48 millimeters into meters

The convert to unit is meter.

Conversion factor written with the converted to unit meter on the top $\dfrac{1 \text{ m}}{1000 \text{ mm}}$

This rate multiplied times the 48 millimeters results with the millimeter unit being canceled as shown below. To divide 48 by 1000 simply move the decimal point three places to the left. 48 millimeters equals 0.048 meters.

$$\frac{1 \text{ m}}{1000 \text{ mm}} \times \frac{48 \text{ mm}}{1} = 0.048 \text{ m}$$

Example 10   Convert 32.9 millimeters into centimeters

The convert to unit is centimeter.

Conversion factor written with the converted to unit centimeter on the top $\dfrac{1 \text{ cm}}{10 \text{ mm}}$

This rate multiplied times the 32.9 millimeters results with the millimeter unit being canceled as shown below. To divide 32.9 by 10 simply move the decimal point one places to the left. 32.9 millimeters equals 3.29 centimeters

$$\frac{1 \text{ cm}}{10 \text{ mm}} \times \frac{32.9 \text{ mm}}{1} = 3.29 \text{ cm}$$

Example 11   Convert 5230 meters into kilometers

The convert to unit is kilometer.

Conversion factor written with the converted to unit kilometer on the top $\dfrac{1 \text{ km}}{1000 \text{ m}}$

This rate multiplied times the 5230 meters results with the meter unit being canceled as shown below. To divide 5230 by 1000 simply move the decimal point three places to the left. 5230 meters equals 5.230 kilometers.

$$\frac{1 \text{ km}}{1000 \text{ m}} \times \frac{5230 \text{ m}}{1} = 5.230 \text{ km}$$

In the United States human heights are commonly written using both the feet and inches units. For instances, a person height is written as 5 feet 10 inches. But in the rest of the world heights are usually measured using either meters or centimeters. For converting between inches and feet and inches measurements the formal dimensional analysis steps will not be written.

Example 12   Sal is 5 feet 11 inches tall. Find his height in centimeter and meters.

First convert from feet inches into inches multiply the feet by 12 and add the inches. In this case multiply the 5 feet times 12 which equal 60 inches then add the 11 inches. As shown below Sal is 71 inches tall.

$$5(12) + 11 = 60 + 11 = 71$$

Now convert 71 inches to centimeter using the conversion factor 2.54 centimeter is equal to 1 inch as shown below with 71 inches equal to 180 centimeters.

$$\frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{71 \text{ in}}{1} \approx 180 \text{ cm}$$

To convert centimeters into meters use the conversion factor 1 meter equals 100 centimeters as shown below which results with 1.80 meters equal to 180 centimeters.

$$\frac{1 \text{ m}}{100 \text{ cm}} \times \frac{180 \text{ cm}}{1} = 1.80 \text{ m}$$

Thus, 5 feet 11 inches is equal to 71 inches which in metric measurements is approximately 180 centimeters or 1.80 meters.

Example 13   Janine is 1.63 meters tall. Find her height in inches.

A direct conversion factor between meters and inches is not listed in this section. Thus, the centimeter unit will serve as the middleman connecting the meter and inch unit. First convert the meters into centimeters then convert the centimeters into inches.   Write the conversion factors 1 inch equals 2.54 centimeters and 100 centimeters equal 2.54 centimeters fraction form to create the expression shown below with the convert to unit inch in the numerator. To evaluate the expression find the product of the denominators 1.63 and 100 by moving the decimal point places to the right and then then write the resulting fraction 163/2.54 in decimal form. So Janine is approximately 64 inches tall.

$$\frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1.63 \text{ m}}{1} = \frac{163}{2.54} \text{ in} \approx 64 \text{ in}$$

To convert Janine's height 64 inches to feet and inches measurement divide 64 by 12 which results in 5 R4 or 5 4/12 which equals 5 feet and 4 inches.

$$
\begin{array}{r}
5\ \text{R}\textbf{4} \\
12\,\overline{)\,\textbf{6 4}} \\
-\underline{6\,0} \\
\textbf{4}
\end{array}
$$

Below some common human heights are written in U.S. customary lengths as feet and inches and inches and in metric lengths as centimeters and meters.

Feet & inches	4' 8"	5' 0"	5' 4"	5' 8"	6' 0"	6' 4"
Inches	56	60	64	68	72	76
Centimeters	142	152	163	173	183	193
Meters	1.42	1.52	1.63	1.73	1.83	1.93

Exercises 6.4

<u>Conversion factors for U.S. customary units of length</u>

12 inches equals 1 foot

3 feet equals 1 yard

1760 yards equals 1 mile

<u>Conversion factors between U.S. customary and metric units of length</u>

1 mile is approximately 1.6 kilometers

1 meter is approximately 3.28 feet

1 inch equals 2.54 centimeters

1-8 Use dimensional analysis to convert between U.S. customary and metric length units using the appropriate conversion factor listed above. *If a direct conversion factor is not listed use multiple conversion factors.*

1.	10 feet into meters	2.	3.5 miles into kilometers
3.	64 inches into centimeters	4.	35 centimeters into inches
5.	10 meters into feet	6.	22 feet into meters
7.	3.2 miles into meters	8.	75 millimeters into inches

9-16   Use dimensional analysis to convert the following lengths within the metric systems using the appropriate conversion factor.

9.	253 centimeters into meters	10.	0.75 meters into millimeters
11.	2,350 meters into kilometers	12.	4.1 kilometers into meters
13.	475 millimeters into meters	14.	5 millimeters into micrometers
15.	300 micrometers into millimeters	16.	0.0023 meters into micrometers

17.   Describe small to large conversion principle in your own words using complete sentence format.

18.   Describe large to small conversion principle in your own words using complete sentence format.

19-26 Perform the following metric conversions using the appropriate conversion principle (small to large OR large to small). Draw a diagram and write a sentence describing the process.

19.   253 centimeters into meters   20.   0.75 meters into millimeters

21.   2,350 meters into kilometers   22.   4.1 kilometers into meters

23.   2.3 meters into centimeters   24.   5 millimeters into micrometers

25.   300 micrometers into millimeters   26.   0.0023 meters into micrometers

27.   The size of a prokaryotic cell is usually between 0.2 and 2.0 micrometers in diameter. Convert both .2 and 2.0 into nanometers.

28.   Visible light has a wavelength between 400 and 750 nanometers. Convert both 400 *nm* and 750 *nm* into meters.

29.   The length of an Ebola virus is approximately 970 nanometers while the length of an E. coli bacterium is about 3 micrometers. Which is larger and approximately how many times larger?