## Section 6.2 Directly proportional relationships

In chapter three, equivalent fractions are created by multiplying the numerator and denominator of a given fraction by the same counting number. This same process also works to create equivalent rates. Suppose that \$5 is the price of 3 jars of a certain brand of jam. What is the price for a case with 12 jars of this jam? To solve create an equivalent rate to the given rate \$5 for 3 jars by multiplying the numerator and denominator by four. As shown below \$20 is the cost of 12 jars of jam.

$$\frac{\$5}{3\text{ jar}} = \frac{\$\,?}{12\text{ jar}} \qquad \overset{\text{Multiply by } 4}{\frac{\$5}{3\text{ jar}} = \frac{\$\,?}{12\text{ jar}}} \qquad \frac{\$5(4)}{3(4)\text{ jar}} = \frac{\$20}{12\text{ jar}}$$

Now, what is the cost of 7 jars of this jam? To solve this problem an equivalent rate to the given rate \$5 of 3 jars is needed with a denominator of 7 jars, but as shown below 3 does not divide evenly into 7.

$$\frac{\$5}{3\text{ jar}} = \frac{\$\,?}{7\text{ jar}}$$

To solve this problem which is known as a proportion usually involves cross multiplication and some algebra techniques. Instead of using an algebra based technique, **dimensional analysis** (which is used to perform conversions later in this chapter) is applied to solve this proportion by creating a product of two fractions.

**To solve directly proportional problems using dimensional analysis**

1. Read the problem carefully and identify the unit of the solution to the problem and label this unit as the output unit.
2. Write the given rate with the output unit in the numerator.
3. Create the fraction product shown below.

$$\overbrace{\left(\frac{\text{output unit}}{\cancel{\text{input unit}}}\right)}^{\text{rate written as a fraction}} \times \frac{\left(\cancel{\text{input unit}}\right)}{1}$$

4. Evaluate the above product which gives the solution to the problem.

The four step dimensional analysis technique to solve directly proportional problems makes the process look more complex than it actually is. The key step is to identify what the problem is asking for and determine the output unit. Once the unit of the solution (the output unit) is determined simply write the given rate with the output unit in the numerator. The follow phrase describes this process, "***write the rate with the output unit on the top***". This dimensional analysis technique not only solves the directly proportional applications in this section, but also conversion problems later in this chapter. Below directly proportional problems are solved using the dimensional analysis technique.

*Example 1* Jerome earns \$12 per 1 hour, find how much he earns if he works 40 hours this week.

The output unit is how much Jerome earns which is measured in dollars.

Rate written as a fraction with the output unit dollar in the numerator $\frac{\$\,12}{1\text{ hr}}$

The above rate multiplied times the 40 hours worked results in hour unit being canceled as shown below. To evaluate this expression simply multiply 12 times 40. Jerome earns \$480 when he works 40 hours.

$$\frac{\$\,12}{1\ \cancel{\text{hr}}} \times \frac{40\ \cancel{\text{hr}}}{1} = \$480$$

*Example 2* Jerome earns \$12 per 1 hour, find how many hours he needs to work in order to earn \$300.

The output unit is how many hours Jerome works which is measured in hours.

Rate written as a fraction with the output unit hour in the numerator $\frac{1\text{ hr}}{\$\,12}$

The above rate multiplied times the 300 dollars earned results in the dollar unit being canceled as shown below. To multiply the fractions first cancel the common factor 3 from 300 and 12 then reduce the resulting fraction 100/4. Jerome needs to work 25 hours to earn \$300.

$$\frac{1\text{ hr}}{\$\,12} \times \frac{\$\,300}{1} = \frac{1\text{ hr}}{\cancel{\$\,12}^{\,4}} \times \frac{\cancel{\$\,300}^{\,100}}{1} = \frac{100}{4}\text{ hr} = 25\text{ hr.}$$

The two previous problems both involved the unit rate \$12 per 1 hour. Notice how the rate is written as a fraction depends on the output units of the problem. In the first problem, the output unit is dollars so the rate is written in fraction from as \$12/1hr with dollars in the numerator. But in the second problem, the output unit is hours so the rate is written in fraction form as 1hr/\$12 with hours in the numerator.

*Example 3* A patient is prescribed 75 milligrams of a medication that is available in a tablet form with 25 mg per 1 tablet. Find the administered dosage measured in tablets for this patient.

The output unit is the dosage given to this patient measured in tablets.

Rate written as a fraction with the output unit tablet in the numerator $\frac{1 \text{ tab}}{25 \text{ mg}}$

This rate multiplied times the 75 milligrams of medication results with the milligram unit being canceled as shown below. To multiply the fractions first cancel the greatest common factor 25 from the 75 and 25. This patient is given three tablets.

$$\frac{1 \text{ tab}}{25 \text{ mg}} \times \frac{75 \text{ mg}}{1} = \frac{1 \text{ tab}}{\cancel{25 \text{ mg}}_{1}} \times \frac{\overset{3}{\cancel{75 \text{ mg}}}}{1} = 3 \text{ tab}$$

*Example 4* A gallon of paint covers approximately 200 square feet. How many gallons of paint are needed to paint 800 square feet of walls.

The output unit is the amount of paint the room requires measured in gallons.

Rate written as a fraction with the output unit gallon in the numerator $\frac{1 \text{ gal}}{200 \text{ sq ft}}$

This rate multiplied times the room size 800 square feet results with the square feet unit being canceled as shown below. To multiply the fractions cancel the common factor 200 from the 800 and 200. For this job, four gallons of paint are needed.

$$\frac{1 \text{ gal}}{200 \text{ sq ft}} \times \frac{800 \text{ sq ft}}{1} = \frac{1 \text{ gal}}{\cancel{200 \text{ sq ft}}_{1}} \times \frac{\overset{4}{\cancel{800 \text{ sq ft}}}}{1} = 4 \text{ gal}$$

The directly proportional problems in the first four examples could have been solved by the creating the following equivalent fractions as shown below.

| **Ex. 1** | **Ex. 2** | **Ex. 3** | **Ex. 4** |
|---|---|---|---|
| $\frac{\$12}{1 \text{ hr}} = \frac{\$ \, ?}{40 \text{ hr}}$ | $\frac{1 \text{ hr}}{\$12} = \frac{? \text{ hr}}{\$300}$ | $\frac{1 \text{ tab}}{25 \text{ mg}} = \frac{? \text{ tab}}{75 \text{ mg}}$ | $\frac{1 \text{ gal}}{200 \text{ sq ft}} = \frac{? \text{ gal}}{800 \text{ sq ft}}$ |

Now, returning to the jam problem stated at the beginning of this section which is solved below using dimensional analysis. For the remaining problems in this section, a calculator is used to evaluate the resulting dimensional analysis expressions.

*Example 5* Given that \$5 is the price of 3 jars of a certain brand of jam, find the price for 7 jars of this jam.

The output unit is the price of 7 jars of jam measured in dollars.

Rate written as a fraction with the output unit dollars in the numerator $\frac{\$5}{3 \text{ jar}}$

This rate multiplied times the size 7 jars results with the jar unit being canceled as shown below. To evaluate, use a calculator to divide 35 by 3. The price of 7 jars of this jam is \$11.67

$$\frac{\$5}{3 \text{ jar}} \times \frac{7 \text{ jar}}{1} = \frac{\$5}{3 \cancel{\text{jar}}} \times \frac{7 \cancel{\text{jar}}}{1} = \frac{\$35}{3} \approx \$11.67$$

*Example 6* Sal drives a compact car 280 miles which uses 9 gallons of gas. Find how far this car travels in miles with a full tank of 14 gallons of gas.

The output unit is the distance the car is driven measured in miles.

Rate written as a fraction with the output unit mile in the numerator $\frac{280 \text{ mi}}{9 \text{ gal}}$

This rate multiplied times the full tank of 14 gallons results with the gallon unit being canceled as shown below. To evaluate use a calculator to multiply 280 times 14, then divide that product 3920 by 9. This car can travel approximately 436 miles with a full tank of 14 gallons.

$$\frac{280 \text{ mi}}{9 \cancel{\text{gal}}} \times \frac{14 \cancel{\text{gal}}}{1} = \frac{3920}{9} \text{ mi} \approx 436 \text{ mi}$$

For a directly proportional relationship to model the above problem it is assumed that this car is driven in similar driving conditions during both the 280 and 436 mile trips resulting in similar gas mileage. Similar driving conditions do not apply if during 280 mile trip the car is driven at 70 miles per hour with the air conditioner on at full blast and during the 436 mile trip the car is driven at 55 miles per hour with the air conditioner off.

*Example 7* On a map, the scale is 250 miles for every 3 inches. Approximately how many miles apart are cities which are 5 inches apart on this map?

The output unit is the distance that the cities are apart in miles.

Rate written as a fraction with the output unit mile in the numerator $\frac{250 \text{ mi}}{3 \text{ in}}$

This rate multiplied times the 5 inches apart results with the inch unit being canceled as shown below. To evaluate use a calculator to multiply 250 times 5, then divide that product 1250 by 3. These cities are approximately 417 miles apart.

$$\frac{250 \text{ mi}}{3 \cancel{\text{in}}} \times \frac{5 \cancel{\text{in}}}{1} = \frac{1250}{3} \text{ mi} \approx 417 \text{ mi}$$

*Example 8* The exchange rate on June 1, 2014 was \$1.32 in U.S. dollars equal to 1 euro, on this date what is the value of \$500 in euros.

The output unit is the value in euros.

Rate written as a fraction with the output unit euro in the numerator $\frac{1 \textit{ Euro}}{\$ 1.32}$

This rate multiplied times the 500 dollars results with the dollar unit being canceled as shown below. To evaluate, use a calculator to divide 500 by 1.32 which results with \$500 equal to approximately 379 Euros on this date.

$$\frac{1 \text{ Euro}}{\cancel{\$} 1.32} \times \frac{\cancel{\$} 500}{1} = \frac{500}{1.32} \text{ Euro} \approx 379 \text{ Euro}$$

*Example 9* The recommended daily dosage for children is often based on the weight of the child. The recommended daily dosage for a certain medication is 12 milligrams per 1 kilogram. Find the recommended daily dosage for a child that weighs 20 kilograms.

The output unit is the daily dosage recommended measured in milligrams.

Rate written as a fraction with the output unit milligram in the numerator $\frac{12 \text{ mg}}{1 \text{ kg}}$

This rate multiplied times the weight of the child 20 kilograms results with the kilogram unit being canceled as shown below. To evaluate this expression simply multiply 12 times 20. The daily dosage for this child is 240 milligrams.

$$\frac{12 \text{ mg}}{1 \cancel{\text{kg}}} \times \frac{20 \cancel{\text{kg}}}{1} = 240 \text{ mg}$$

## Exercises 6.2

1-13 Solve the following directly proportional problems. **Show the dimensional analysis calculation.**

1. The average pay for an auto mechanic is $20 per 1 hour. Find how much an average mechanic earns when working a 40 hour week.

2. The price of 5 cans of store brand corn is $8.00. Find the price of 20 cans of this store brand corn.

3. The estimated gas mileage of a new hybrid car is 42 miles per 1 gallon. Find how many miles can this car be driven with a full tank of 15 gallons of gas.

4. One gallon of paint covers approximately 200 square feet. Find how many square feet can be painted with 3.5 gallons of paint.

5. On a map, the scale is 1 inch represents 50 miles. Find how many miles apart are two cities which are 4.5 inches apart on the map.

6. Bill types at the rate of 3 pages every 20 minutes. Find the time it would take Bill to type a 12 page term paper.

7. Sarah's watch loses time at the rate of 3 minutes every 5 days. Find the number days before the watch loses one hour of time.

8. The odds of a certain wager are $3 won for every $2 bet. Find the winning from a successful $24 bet.

9. A patient is prescribed 200 milligrams of a medication that is available in a tablet form with 50 milligrams per 1 tablet. Find the number of tablets given to this patient.

10. A patient is prescribed 120 milligrams of a medication that is available in a tablet form with 80 milligrams per 1 tablet. Find the number of tablets given to this patient.

11. The recommended daily dosage for a certain medication is 8 milligrams of medicine per every 1 kilogram of patient weight, what is the recommended daily dosage in milligrams for a patient that weighs 70 kilograms.

12. The standard TV display is 4 inches wide for every 3 inches high. Find the height of the standard display on a 32 inch wide TV.

13. A widescreen TV display is 16 inches wide for every 9 inches high. Find the height of the widescreen display on a 32 inch wide TV.

14-27 Solve the following directly proportional problems. **Show the dimensional analysis calculation.** Use a calculator to evaluate the resulting expression.

14. The average pay for a registered nurse at a local hospital is $40 per 1 hour. Find the hours a R.N. at that hospital works in order to earn $1540.

15. The price of 5 cans of store brand corn is $8.00. Find the price of 7 cans of this store brand corn.

16. The estimated gas mileage of a given new car is 32 miles per 1 gallon. Estimate the number of gallons consumed when this car is driven 384 miles.

17. The estimated gas mileage of a given new hybrid car is 45 miles per 1 gallon on the highway. Estimate total distance in miles this car can be driven on a highway with a full 14.2 gallon tank.

18. One gallon of paint covers approximately 200 square feet. Find the gallons of paint needed to paint 540 square feet?

19. On a map, the scale is 300 kilometers for every 4 centimeters. Approximately how many kilometers apart are two cities that are 7.5 centimeters apart on this map?

20. Carlos jogs 3 miles in 20 minutes. At that rate, how long will it take Carlos to jog 10 miles?

21. The exchange rate in April 2013 was 12.34 Mexican pesos for 1 U.S. dollar. Find the value of $400 in pesos on this date.

22. The exchange rate in April 2013 was 40.80 Philippines pesos for 1 U.S. dollar. Find the value of 1000 pesos in dollars on this date.

23. The exchange rate in April 2013 was 0.98 Canadian dollars for 1 U.S. dollar. Find the value of 500 Canadian dollars in U.S. dollars on this date.

24. The standard TV display is 4 inches wide for every 3 inches high. Find the height of the standard display on a 42 inch wide TV.

25. A widescreen TV display is 16 inches wide for every 9 inches high. Find the height of the widescreen display on a 42 inch wide TV.

26. The speed of light is approximately 300,000 kilometers per 1 second. The minimum distance between the earth and the sun is 147,000,000 kilometers. Estimate the time in seconds that is takes light from the sun to reach earth.

27. The health department is predicting that during this flu season 58 people will become infected with the flu out of every 1000 people. Estimate the number infected with the flu in Fairfield whose population is approximately 105,000.