

Section 5.4 Percent and fraction formats

A percent means per hundred with the percent symbol % indicating a fraction with a denominator of one hundred. A percent is written as a decimal by removing the percent symbol % and moving the decimal point two places to the left and for the reverse procedure a decimal is written as a percent by inserting a percent symbol % and moving the decimal point two places to the right. In this section, decimals will serve as the middleman when converting between percent and fraction formats.

To convert from percent to fraction format

First convert the percent to decimal format by removing the percent symbol % and moving the decimal point two places to the left. Then write the resulting decimal as a fraction and reduce if possible.

Example 1 Write 6%, 84% and 3.5% as fractions in reduced form.

First write 6% in decimal format by removing the percent sign and moving the decimal point two places to the left. Then write the resulting decimal 0.06 as the fraction $\frac{6}{100}$ which reduces to $\frac{3}{50}$. 6% can be visualized as either the rate 6 out of 100 or 3 out of 50.

$$6\% = 0.06 = \frac{6}{100} = \frac{(3)(\cancel{2})}{(50)(\cancel{2})} = \frac{3}{50}$$

First write 84% in decimal format by removing the percent sign and moving the decimal point two places to the left. Then write the resulting decimal 0.84 as the fraction $\frac{84}{100}$ which reduces to $\frac{21}{25}$. 84% can be visualized as either the rate 84 per every 100 or 21 per every 25.

$$84\% = 0.84 = \frac{84}{100} = \frac{(21)(\cancel{4})}{(25)(\cancel{4})} = \frac{21}{25}$$

First write 3.5% in decimal format by removing the percent sign and moving the decimal point two places to the left. Then write the resulting decimal 0.035 as the fraction $\frac{35}{1000}$ which reduces to $\frac{7}{200}$. 3.5% can be visualized as either the rate 3.5 out of 100, 35 out of 1000, or 7 out of 200.

$$3.5\% = 0.035 = \frac{35}{1000} = \frac{(7)(\cancel{5})}{(200)(\cancel{5})} = \frac{7}{200}$$

For a percent larger than 100%, the resulting decimal is larger than one and in fraction form is written either as an improper fraction or as a mixed number.

Example 2 Write 213% and 140% as fractions.

First write 213% in decimal format by removing the percent sign and moving the decimal point two places to the left. Then write the resulting decimal 2.13 as the fraction $\frac{213}{100}$ which is in reduced form. 213% can be visualized as the rate 213 per every 100.

$$213\% = 2.13 = 2\frac{13}{100} \text{ or } \frac{213}{100}$$

First write 140% in decimal format by removing the percent sign and moving the decimal point two places to the left. Then write the resulting decimal 1.40 as the fraction $\frac{140}{100}$ which reduces to $\frac{7}{5}$. 140% can be visualized as either the rate 140 per every 100 or 7 per every 5.

$$140\% = 1.40 = 1\frac{40}{100} = 1\frac{\cancel{40}^2}{\cancel{100}_5} = 1\frac{2}{5} \text{ or } \frac{7}{5}$$

In the previous problems when converting a percent to fraction format, the percent is first written as a decimal then the decimal is written as a fraction. When reversing this process by converting a fraction into a percent, decimals are again used as the middleman with a fraction first written as a decimal then the decimal is written as a percent. As a reminder, fractions are converted into a decimal format by dividing the denominator into the numerator using long division. Below the fraction $\frac{5}{8}$ is first converted into decimal format by dividing the denominator 8 into the numerator 5. Then, the resulting decimal 0.625 is written in percent format as 62.5% by inserting the % symbol and moving the decimal point two places to the right.

$\frac{5}{8} = 0.625 = 62.5\%$	$\begin{array}{r} .625 \\ 8 \overline{) 5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$	<p><i>Line up the decimal places</i></p> <p><i>Write the dividend 5 as 5.000</i></p> <p><i>6 times 8 equals 48</i></p> <p><i>Drop a zero digit</i></p> <p><i>2 times 8 equals 16</i></p> <p><i>Drop a zero digit</i></p> <p><i>5 times 8 equals 40</i></p> <p><i>0 remainder terminating decimal</i></p>
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To convert from fraction to percent format

First convert the fraction to decimal format by using long division. Then write the resulting decimal as a percent by inserting the percent symbol % and moving the decimal point two places to the right.

Example 3 Write the fraction $8/25$ as a percent.

The fraction $8/25$ is converted into decimal format by dividing the denominator 25 into the numerator 8. The resulting decimal 0.32 is written in percent format as 32% by inserting the % symbol and moving the decimal point two places to the right.

$$\frac{8}{25} = 0.32 = 32\%$$

25	$\begin{array}{r} .32 \\ \overline{)8.00} \\ -75 \\ \hline 50 \\ -50 \\ \hline 0 \end{array}$	<p><i>Line up the decimal places</i></p> <p><i>Write the dividend 8 as 8.00</i></p> <p><i>3 times 25 equals 75</i></p> <p><i>Drop a zero digit</i></p> <p><i>2 times 25 equals 50</i></p> <p><i>0 remainder terminating decimal</i></p>
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Example 4 Write the fraction $3/7$ as a percent rounded to one decimal place.

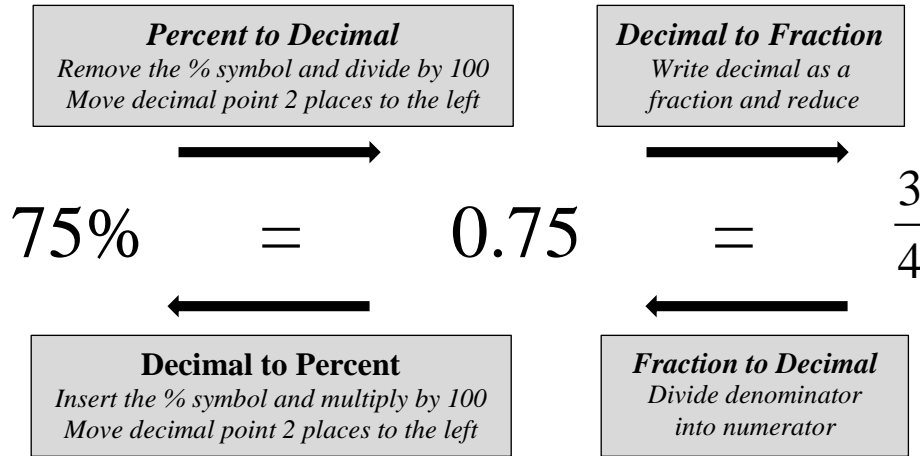
The fraction $3/7$ is converted into decimal format by dividing the denominator 7 into the numerator 3. The resulting decimal is rounded to 0.429 and is written in percent format as 42.9% by inserting the % symbol and moving the decimal point two places to the right.

$$\frac{3}{7} \approx 0.429 = 42.9\%$$

7	$\begin{array}{r} .4285 \\ \overline{)3.0000} \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 40 \\ \hline 35 \\ \hline 5 \end{array}$	<p><i>Line up the decimal places</i></p> <p><i>Write the dividend 3 as 3.0000</i></p> <p><i>4 times 7 equals 28</i></p> <p><i>Drop a zero digit</i></p> <p><i>2 times 7 equals 14</i></p> <p><i>Drop a zero digit</i></p> <p><i>8 times 7 equals 56</i></p> <p><i>Drop a zero digit</i></p> <p><i>5 times 7 equals 35</i></p> <p><i>Round 0.4285 to 3 decimal places</i></p>
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0.4285 rounded to three decimal places is 0.429

The diagram below displays the relationship between the percent, decimal, and fraction formats. Notice how the decimal format serves as the middleman between the percent and fraction formats.



Below halves, thirds, fourths, fifths and tenths are written in both fraction and percent formats. If possible, try to memorize these relationships which will come in handy when working with percentages.

Halves	$\frac{1}{2} = 50\%$			
Thirds	$\frac{1}{3} \approx 33\%$	$\frac{2}{3} \approx 67\%$		
Fourths	$\frac{1}{4} = 25\%$	$\frac{2}{4} = 50\%$	$\frac{3}{4} = 75\%$	
Fifths	$\frac{1}{5} = 20\%$	$\frac{2}{5} = 40\%$	$\frac{3}{5} = 60\%$	$\frac{4}{5} = 80\%$
Tenths	$\frac{1}{10} = 10\%$	$\frac{2}{10} = 20\%$	$\frac{3}{10} = 30\%$	$\frac{4}{10} = 40\%$
	$\frac{5}{10} = 50\%$	$\frac{6}{10} = 60\%$	$\frac{7}{10} = 70\%$	$\frac{8}{10} = 80\%$
				$\frac{9}{10} = 90\%$

Example 5 Fill in the following table

Convert 28% to a decimal by removing the percent symbol and moving the decimal point two places to the left. Write the resulting decimal 0.28 as a fraction and reduce.

$$28\% = 0.28 = \frac{\cancel{28}^7}{\cancel{100}_{25}} = \frac{7}{25}$$

Convert 0.45 to a percent by inserting the percent symbol and moving the decimal point two places to the right. Write the decimal 0.45 as the fraction 45/100 and reduce.

$$45\% = 0.45 = \frac{\cancel{45}^9}{\cancel{100}_{20}} = \frac{9}{20}$$

Recognize the fraction 3/4 as the decimal 0.75 which equals 75%.

$$75\% = 0.75 = \frac{3}{4}$$

Convert 120% to a decimal by removing the percent symbol and moving the decimal point two places to the left. Write the resulting decimal 1.20 as the fraction 1 20/100 and reduce.

$$120\% = 1.20 = 1\frac{\cancel{20}^1}{\cancel{100}_5} = 1\frac{1}{5} \text{ or } \frac{6}{5}$$

Write the fraction 8/11 as a decimal using long division. The resulting decimal when rounded to two decimal places is 0.73 which equals 73%

$$73\% = 0.73 \approx \frac{8}{11}$$

Percent	Decimal	Fraction
28%		
	0.45	
		$\frac{3}{4}$
120%		
		$\frac{8}{11}$

Percent	Decimal	Fraction
28%	0.28	$\frac{7}{25}$
45%	0.45	$\frac{9}{20}$
75%	0.75	$\frac{3}{4}$
120%	1.20	$\frac{6}{5}$
73%	0.73	$\frac{8}{11}$

$$\begin{array}{r}
 .727 \\
 11 \overline{) 8.000} \\
 \underline{-77} \\
 30 \\
 \underline{-22} \\
 80 \\
 \underline{-77} \\
 3
 \end{array}$$

Line up the decimal places
Write the dividend 8 as 8.000
7 times 11 equals 77
Drop a zero digit
2 times 11 equals 22
Drop a zero digit
7 times 11 equals 77
Repeating decimal

0.727 rounded to two decimal places 0.73

Exercises 5.4

1-9 Write the following percentages as fractions in reduced form.

1. 35%

2. 40%

3. 25%

4. 8%

5. 3.5%

6. 38%

7. 250%

8. 125%

9. 16.7%

10-15 Write the following fractions in percent format. Show the long division.

10. $\frac{11}{20}$

11. $\frac{7}{8}$

12. $\frac{9}{25}$

13. $\frac{37}{40}$

14. $1\frac{9}{40}$

15. $2\frac{7}{20}$

For #16-24 round decimals to 3 decimal places and the percents to 1 decimal place

16-24 Write the following fractions in percent format. Show the long division.

16. $\frac{3}{7}$

17. $\frac{1}{12}$

18. $\frac{7}{11}$

19. $\frac{2}{9}$

20. $\frac{4}{13}$

21. $\frac{5}{14}$

22. $\frac{1}{16}$

23. $1\frac{5}{6}$

24. $2\frac{1}{12}$

25-33 Write the following fractions in percent format (without showing steps)

25. $\frac{3}{4}$

26. $\frac{1}{2}$

27. $\frac{2}{5}$

28. $\frac{7}{10}$

29. $\frac{2}{3}$

30. $\frac{1}{4}$

31. $\frac{3}{10}$

32. $\frac{1}{5}$

33. $\frac{3}{5}$

34-43 Fill in the following tables.

34.

Percent	Decimal	Fraction
35%		

35.

Percent	Decimal	Fraction
	0.32	

36.

Percent	Decimal	Fraction
		$\frac{11}{40}$

37.

Percent	Decimal	Fraction
140%		

38.

Percent	Decimal	Fraction
	0.05	

39.

Percent	Decimal	Fraction
		$\frac{4}{5}$

40.

Percent	Decimal	Fraction
8.5%		

41.

Percent	Decimal	Fraction
	0.004	

42.

Percent	Decimal	Fraction
		$\frac{7}{25}$

43.

Percent	Decimal	Fraction
	0.305	