Section 4.2   Rounding and comparing decimals

One advantage of the decimal format in comparison to the fraction format is that when evaluating operations involving decimals the procedures are similar to those used with whole numbers. In this section, decimals are rounded using the same technique that is employed in the first chapter to round whole numbers.

To round a whole number or a decimal to a given place value

Case 1   If the digit to the immediate right of the rounded to place value is less than 5 the digit in the rounded to place value remains the same and all the digits to the right of the rounded to place value are changed to 0 digits.

Case 2   If the digit to the immediate right of the rounded to place value is 5 or larger the digit in the rounded to place value is increased by 1 and all the digits to the right of the rounded to place value are changed to 0 digits.

To round a whole number or decimal to a given place value first locate the digit in the rounded to place value then circle it (*bolded and boxed in the textbook*) and draw a bar behind that digit. If the digit to the immediate right of the rounded to digit place is 5 or larger the digit in the rounded to place value is increased by 1. The digits to the right of the rounded to place value indicated by the bolded line are changed to 0 digits. For decimals these ending zeros to the right of the rounded to place value are not written as part of the final answer.

Example 1   Round the following numbers to the specified place value.

Round 85,**6**39 to the **hundreds** place

To the immediate right of the rounded to **hundreds** place is the digit 3 which is less than 5 so the digit in the **hundreds** place 6 stays the same and all the digits to the right of the **hundreds** place are changed to zeros.
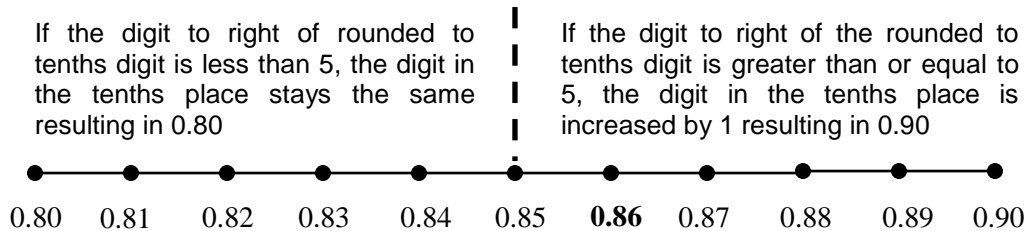
$$8\ \ 5\ ,\ \boxed{\mathbf{6}}\ \overset{0}{\underset{}{3}}\ \overset{0}{\underset{}{9}} \qquad\qquad\qquad 85,\mathbf{6}00$$

Round 0.2**3**68 to **hundredths** place

To the immediate right of the rounded to **hundredths** place is the digit 6 which is larger than or equal to 5 so the digit in the **hundredths** place is increased by 1 from 3 to 4 and all the digits to the right of the **hundredths** place are changed to zeros.

$$0\ .\ 2\ \overset{\mathbf{1}}{\boxed{\mathbf{3}}}\ \overset{0}{6}\ \overset{0}{8} \qquad\qquad\qquad 0.2\mathbf{4}$$

The process of rounding a decimal is visually illustrated below with a number line. To round 0.86 to the tenths place, the decimal 0.85 is plotted halfway between 0.80 and 0.90 on the number line and serves as the cutoff point when rounding to the nearest tenth. The decimal 0.86 rounded to the tenths place is equal to 0.90, since 0.86 is larger than 0.85 and therefore closer to 0.90

If the digit to right of rounded to tenths digit is less than 5, the digit in the tenths place stays the same resulting in 0.80

If the digit to right of the rounded to tenths digit is greater than or equal to 5, the digit in the tenths place is increased by 1 resulting in 0.90

0.80   0.81   0.82   0.83   0.84   0.85   **0.86**   0.87   0.88   0.89   0.90

Example 2   Round the following numbers to the specified place value.

Round 0.02**8**736 to the nearest **thousandth**

To the immediate right of the rounded to **thousandths** place is the digit 7 which is larger than or equal to 5 so the digit in the **thousands** place is increased by 1 from 8 to 9 and all the digits to the right of the **thousandths** place are changed to zeros. The ending zeros are not written in the final answer.

$$0\ .\ 0\ 2\ \boxed{8}\ \cancel{7}\ \cancel{3}\ \cancel{6}$$

with **1** | 0 0 0

0.02**9**

Round 13.**1**29 to the nearest **tenth**

To the immediate right of the rounded to **tenths** place is digit 2 which is less than 5 so the digit in the **tenths** place 1 stays the same and all the digits to the right of the **tenths** place are changed to zeros and are not written in the final answer.

$$1\ \ 3\ .\ \boxed{1}\ \cancel{2}\ \cancel{9}$$

with | 0 0

13.**1**

Round 4.2**9**8 to two decimal places

To the immediate right of the rounded to **hundredths** place is digit 8 which is larger than 5 so the digit in the **hundreds** place is increased by 1 which changes 29 to 30 and all the digits to the right of the **hundredths** place are changed to zeros.

$$4\ .\ 2\ \boxed{9}\ \cancel{8}$$

with **1** | 0

4.3**0**

In the next example, the same number is rounded to different place values which results in different answers. Notice how the answers depend on which place the number is rounded to.

Example 3   Round 39.295 to the tens, ones, tenths, and hundredths place.

Round **3**9.295 to the **tens** place

To the immediate right of the rounded to **tens** place is the digit 9 which is larger than or equal to 5 so the digit in the **tenths** place is increased by 1 from 3 to 4 and all the digits to the right of the **tens** place are changed to zeros. The ending zeros are not written in the final answer.

1 0   0   0   0
3 9 . 2 9 5   **4**0

Round 3**9**.295 to the **ones** place

To the immediate right of the rounded to **ones** place is the digit 2 which is less than 5 so the digit in the **ones** place 9 stays the same and all the digits to the right of the **ones** place are changed to zeros with the ending zeros not written in the final answer.

0 0 0
3 **9** . 2 9 5   3**9**

Round 39.**2**95 to the **tenths** place

To the immediate right of the rounded to **tenths** place is the digit 9 which is larger than or equal to 5 so the digit in the **tenths** place is increased by 1 from 2 to 3 and all the digits to the right of the **tenths** place are changed to zeros. The ending zeros are not written in the final answer.

1 0 0
3 9 . **2** 9 5   39.**3**

Round 39.2**9**5 to the **hundredths** place

To the immediate right of the rounded to **hundredths** place is the digit 5 which is equal to 5 so the digit in the **hundredths** place is increased by 1 which changes 29 to 30 and all the digit to the right of the **hundredths** place are changed to zero.

1 0
3 9 . 2 **9** 5   39.3**0**

When rounding money the same procedure applies. Money is often rounded to either the nearest dollar or the nearest cent. Rounding to the nearest dollar means rounding to the nearest ones place while rounding to the nearest cent means rounding to the nearest hundredths place.

Example 4   Round the list price of a tablet computer $874.99 to the nearest dollar

Rounding $87**4**.99 to nearest **dollar** means rounding to the nearest **ones** place. As shown below the rounded cost of this tablet computer is $875

$$8\quad7\quad\boxed{4}\overset{\mathbf{1}\ \big|\ 0\ 0}{.9\ 9}$$

$875

Example 5   Round the list price of a new car $16,999.53 to the nearest dollar.

Rounding $16,99**9**.53 to nearest **dollar** means rounding to the nearest **ones** place. As shown below, the rounded cost of this new car to the nearest dollar is $17,000

$$1\quad6\ ,\ 9\quad9\quad\boxed{9}\overset{\mathbf{1}\ \big|\ 0\ 0}{.5\ 3}$$

$17,000

Example 6   At the local service station the price of gasoline is $3.869 per gallon. Round to the nearest cent the price of gasoline per gallon.

Rounding $3.8**69** to nearest **cent** means rounding to the nearest **hundredth** place. As shown below, the rounded cost of gasoline to the nearest cent is $3.87 per gallon.

$$3\ .\ 8\ \boxed{6}\overset{\mathbf{1}\ \big|\ 0}{\ 7}$$

$3.87

Example 7   On January 1, 2013 the official exchange rate listed one Chinese Yuan being worth 0.1586 in United States Dollars. Round to the nearest cent the value of one Yuan on that date.

Rounding $0.1**5**86 to the nearest cent means rounding to the nearest **hundredths** place. As shown below on January 1, 2013 a Chinese Yuan was valued at approximately sixteen cents which is written $0.16

$$0\ .\ 1\ \boxed{5}\overset{\mathbf{1}\ \big|\ 0\ 0}{8\ 6}$$

$0.16

Definition   **Like decimals** are decimals that have the same number of decimal places and thus when written in fraction form are like fractions.

Comparing decimals is a simple process. If two proper decimals are like decimals determining which is larger can be done mentally without showing any steps since they are like fractions and for like fractions the larger numerator determines which is larger. To compare unlike decimals that have different numbers of decimal places, zero digits are placed at the end of the shorter decimal so that both decimals have the same number of decimal places.

Example 8   Insert the appropriate symbol $<$ or $>$ to make a true statement

0.37 __ 0.41   0.37 $<$ 0.41

37 hundredths and 41 hundredths are like decimals with 37/100 smaller than 41/100

0.235 __ 0.219   0.235 $>$ 0.219

235 thousandths and 219 thousandths are like decimals with 235/1000 larger than 219/1000

0.7 __ 0.69   0.7**0** __ 0.69   0.7**0** $>$ 0.69

First create like decimals by adding ending zeros so that all the decimals have the same ending decimal place in this case the hundredths place. 70 hundredths and 69 hundredths are like decimals with 70/100 larger than 69/100

0.539 __ 0.54   0.539 __ 0.54**0**   0.539 $<$ 0.54**0**

First create like decimals by adding ending zeros so that all the decimals have the same ending decimal place in this case the thousandths place. 539 thousandths and 540 thousandths are like decimals with 539/1000 smaller than 540/1000

3.2 __ 3.25   3.2**0** __ 3.25   3.2**0** $<$ 3.25

First create like decimals by adding ending zeros so that all the decimals have the same ending decimal place in this case the hundredths place. Three and twenty hundredths and three and twenty five hundredths are like decimals with 3 20/100 smaller than 3 25/100

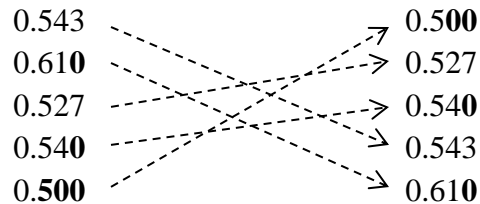6.01 __ 5.9   6.01 $>$ 5.9

Since the whole numbers parts are different there is no need to create like decimals. The whole number in the first mixed decimal is larger so it is the larger number.

In the problems below the same technique used to compare two decimals is extended to compare a list of decimals. To write a list of decimals from smallest to largest first create like decimals by adding ending zeros as necessary such that all the decimals have the same number of decimals places.
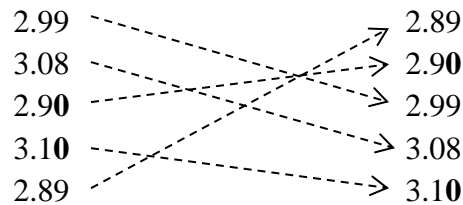
Example 9   List from smallest to largest 0.543, 0.61, 0.527, 0.54 and 0.5

First create like decimals by adding ending zeros as needed so that all the decimals have the same ending decimal place, in this case the thousandths place. Then list their numerators from smallest to largest 500, 527, 540, 543 and 610. As shown below 0.500, 0.527, 0.540, 0.543 and 0.610 are listed from smallest to largest.

$$
\begin{array}{ll}
0.543 & 0.5\mathbf{00} \\
0.61\mathbf{0} & 0.527 \\
0.527 & 0.54\mathbf{0} \\
0.54\mathbf{0} & 0.543 \\
0.\mathbf{500} & 0.61\mathbf{0}
\end{array}
$$

Example 10   List from smallest to largest 2.99, 3.08, 2.9, 3.1 and 2.89

First create like decimals by adding ending zeros as needed so that all the decimals have the same ending decimal place, in this case the hundredths place. First compare the three decimals with the same whole number part 2 and list their numerators from smallest to largest 89, 90 and 99. Then compare the two decimals with the same whole number part 3 and list their numerators from smallest to largest 8 and 10. As shown below 2.89, 2.90, 2.99, 3.08 and 3.10 are listed from smallest to largest.

$$
\begin{array}{ll}
2.99 & 2.89 \\
3.08 & 2.9\mathbf{0} \\
2.9\mathbf{0} & 2.99 \\
3.1\mathbf{0} & 3.08 \\
2.89 & 3.1\mathbf{0}
\end{array}
$$

Example 11   Find the median time in seconds for the runners in the men's 100 meters medal race whose finishing times are listed below.

<div align="center">

9.8   9.79   9.63   9.88   9.75   9.98   9.94

</div>

List the seven times in order from smallest to largest and then cross out two numbers at a time the smallest and largest remaining times on the list until only the middle number 9.80 is left. The median finishing time for this race is 9.80 seconds.

<div align="center">

~~9.63~~   ~~9.75~~   ~~9.79~~   9.8**0**   ~~9.88~~   ~~9.94~~   ~~9.98~~

</div>
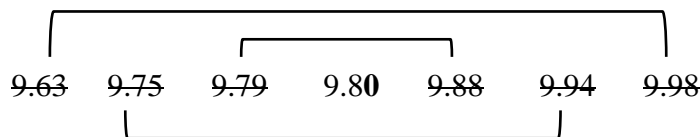
Exercises 4.2

1-10   Round the following numbers to the indicated place.

1.   Round 45,981 to the nearest hundred

2.   Round 8732 to the nearest ten

3.   Round 2.895 to the nearest tenth

4.   Round 0.5642 to the nearest hundredth

5.   Round 0.01259 to the nearest thousandth

6.   Round 0.05978 to the nearest thousandth

7.   Round 0.00045658 to the nearest millionth

8.   Round 0.0000823 to the nearest millionth

9.   Round 0.1923 to two decimal places

10.   Round 0.321578 to three decimal places

11-18   Round the following numbers to the nearest indicated places.

11.	8953	hundred	ten	
12.	12,549	hundred	ten	
13.	23.06	ten	one	tenth
14.	121.965	ten	one	tenth
15.	3.0293	tenth	hundredth	thousandth
16.	0.5008	tenth	hundredth	thousandth
17.	0.2586	1 decimal place	2 decimal places	3 decimal places
18.	4.89267	1 decimal place	2 decimal places	3 decimal places

19-24   Round the following to the nearest dollar

19.	$123.89	20.	$19.76	21.	$275.49
22.	$12,003.08	23.	$9999.79	24.	$199.39

25-30   Round the following to the nearest cent

25.	$4.059	26.	$3.459	27.	$0.054
28.	$0.173	29.	$1.035	30.	$2.099

31-39 Insert the appropriate symbol < or > to make a true statement.

31.   0.28   0.19   32.   0.78   0.81   33.   0.053   0.1

34.   0.02   0.007   35.   0.35   0.349   36.   5.37   5.4

37.   7.59   7.6   38.   2.9   3.01   39.   4.89   3.95

40-45 Write the following list of numbers from smallest to largest.

40.   0.872, 0.8, 0.869, 0.88 and 0.799   41.   0.23, 0.219, 0.2, 0.225 and 0.209

42.   0.913, 0.92, 0.907, 0.9 and 0.95   43.   0.01, 0.1, 0.121, 0.12 and 0.101

44.   4.001, 3.9, 4.01, 3.99 and 3.989   45.   7.324, 6.7, 7.4, 6.68 and 7.35

46-47 Find the median of the following numbers.

46.   0.83   0.812   0.825   0.9   0.831

47.   12.9   13.05   12.84   12.98   13.017   13.1   12.895