Section 3.1 Least Common Multiple

In this section, the concept of the least common multiple is introduced and three techniques to find the least common multiple are described. In this chapter, the main topic is addition and subtraction of fractions which involves finding a least common multiple when the denominators are different.

The **least common multiple (LCM)** of two or more natural numbers is the least (smallest) number which is a multiple of all the numbers. In other words, the LCM is the least (smallest) number which is divisible by all the given numbers.

To understand the concept of the least common multiple, first consider the least common multiple of 6 and 9. The LCM of 6 and 9 is a multiple that is common to both 6 and 9 and it is the least (smallest) of these common multiples. Below the multiples of 6 and 9 are listed separately with the common multiples bolded. Notice that 18 is the least (smallest) of the common multiples 18, 36, 54, 72, … and thus 18 is the least (smallest) number that is divisible by both 9 and 18.

6, 12, **18**, 24, 30, **36**, 42, 48, **54**, 60, 66, **72**, 78, …

9, **18**, 27, **36**, 45, **54**, 63, **72**, 81, 90, 99, …

To shortcut this process of finding the LCM of 6 and 9 instead of listing the multiplies of both 6 and 9 until a common multiple is identified simply list the multiplies of the larger number 9 only until a resulting multiple in this case 18 is also divisible by the smaller number 6 as shown below.

9, **18**

To find the LCM by listing multiples of the larger number

1) Create a list of multiples of the largest number.

2) Stop listing multiples of the larger number as soon one of these multiples is divisible by all the other numbers. This last multiple on the list is the LCM.

Example 1 Find the LCM by listing the multiples of the largest number.

LCM of 8 and 10 10, 20, 30, **40**

To find the LCM of 8 and 10 list the multiples of the larger number 10 until a resulting multiple is divisible by the smaller number 8. The LCM of 8 and 10 is 40.

LCM of 4 and 6 6, **12**

To find the LCM of 4 and 6 list the multiples of the larger number 6 until a resulting multiple is divisible by the smaller number 4. The LCM of 4 and 6 is 12.

LCM of 9 and 15 15, 30, **45**

To find the LCM of 9 and 15 list the multiples of the larger number 15 until a resulting multiple is divisible by the smaller number 9. The LCM of 9 and 15 is 45.

LCM of 3 and 9 **9**

To find the LCM of 3 and 9 list the multiples of the larger number 9 until a resulting multiple is divisible by the smaller number 3. Note since 9 is divisible by 3 no other multiples need to be listed. The LCM of 3 and 9 is 9.

LCM of 5, 6 and 12 12, 24, 36, 48, **60**

To find the LCM of 5, 6 and 12 list the multiples of the largest number 12 until a resulting multiple is divisible by both the other numbers 5 and 6. The LCM of 5, 6 and 12 is 60.

The listing of multiples method to find least common multiples can become a tedious and inefficient process when the LCM is a large number. For instance, the least common multiple of 38 and 46 is 874, but most of us could not mentally list the multiples of 46 and determine which are divisible by 38 until the LCM 874 is found. Another technique based on prime factorization is more efficient in finding least common multiples which are not readily found by simply listing a few multiples of the larger number. The definition of prime factorization is restated below.

Prime factorization is the process of writing a composite number as a product consisting of only prime factors.

Find the LCM using the prime factorization method

1) Write each number is prime factorization form.

2) The LCM is the product formed by using all the different prime number divisors listed in the prime factorizations as factors with each prime divisor written as a factor only once and raised to the highest exponent to which it is raised in any of the prime factorizations.

3) To find the LCM multiply the product found in the above step.

Example 2 Find the LCM of 36 and 40 using the prime factorization method.

First write 36 and 40 in prime factorization form. The primes 2, 3 and 5 are factors of the LCM. To create the LCM $(2)^3(3)^2(5)$ the prime 2 is raised to the third power and prime 3 is raised to the second power since those are the highest powers that these primes are raised to in the prime factorizations. The LCM of 36 and 40 is 360 as shown below.

$$\begin{array}{r|l} \mathbf{2} & 36 \\ \hline \mathbf{2} & 18 \\ \hline \mathbf{3} & 9 \\ \hline & \mathbf{3} \end{array} \qquad \begin{array}{r|l} \mathbf{2} & 40 \\ \hline \mathbf{2} & 20 \\ \hline \mathbf{2} & 10 \\ \hline & \mathbf{5} \end{array}$$

$36 = (\mathbf{2})^2(\mathbf{3})^2$
$40 = (\mathbf{2})^3(\mathbf{5})$
LCM $(\mathbf{2})^3(\mathbf{3})^2(\mathbf{5})$
$360 = (8)(9)(5)$

Example 3 Find the LCM of 38 and 46 using the prime factorization method.

First write 38 and 46 in prime factorization form. The primes 2, 19 and 23 are factors of the LCM. To create the LCM (2)(19)(23) the repeated prime divisor 2 is listed only once. The LCM of 38 and 46 is 874 as shown below.

$$\begin{array}{r|l} \mathbf{2} & 38 \\ \hline & \mathbf{19} \end{array} \qquad \begin{array}{r|l} \mathbf{2} & 46 \\ \hline & \mathbf{23} \end{array}$$

$38 = (\mathbf{2})(\mathbf{19})$
$46 = (\mathbf{2})(\mathbf{23})$
LCM $(\mathbf{2})(\mathbf{19})(\mathbf{23})$
$874 = (38)(23)$

Example 4 Find the LCM of 9, 30 and 35 using the prime factorization method.

First write 9, 30 and 35 in prime factorization form. The primes 2, 3, 5 and 7 are factors of the LCM. To create the LCM $(2)(3)^2(5)(7)$ the prime 3 is raised to the second power since that is the highest power that 3 is raised to in the prime factorizations and the repeated prime divisor 5 is listed only once. The LCM of 9, 30 and 35 is 630 as shown below

$$\begin{array}{r|l} \mathbf{3} & 9 \\ \hline & \mathbf{3} \end{array} \qquad \begin{array}{r|l} \mathbf{2} & 30 \\ \hline \mathbf{3} & 15 \\ \hline & \mathbf{5} \end{array} \qquad \begin{array}{r|l} \mathbf{5} & 35 \\ \hline & \mathbf{7} \end{array}$$

$$\begin{aligned} 9 &= (\mathbf{3})^2 \\ 30 &= (\mathbf{2})(\mathbf{3})(\mathbf{5}) \\ 35 &= (\mathbf{5})(\mathbf{7}) \\ \text{LCM} &\quad (\mathbf{2})(\mathbf{3})^2(\mathbf{5})(\mathbf{7}) \\ 630 &= (2)(9)(5)(7) \end{aligned}$$

Example 5 Find the LCM of 10, 12 and 14 using the prime factorization method.

First write 10, 12 and 14 in prime factorization form. The primes 2, 3, 5 and 7 are factors of the LCM. To create the LCM $(2)^2(3)(5)(7)$ the prime 2 is raised to the second power since that is the highest power that the prime 2 is raised to in the prime factorizations. The LCM of 10, 12 and 14 is 420 as shown below.

$$\begin{array}{r|l} \mathbf{2} & 10 \\ \hline & \mathbf{5} \end{array} \qquad \begin{array}{r|l} \mathbf{2} & 12 \\ \hline \mathbf{2} & 6 \\ \hline & \mathbf{3} \end{array} \qquad \begin{array}{r|l} \mathbf{2} & 14 \\ \hline & \mathbf{7} \end{array}$$

$$\begin{aligned} 10 &= (\mathbf{2})(\mathbf{5}) \\ 12 &= (\mathbf{2})^2(\mathbf{3}) \\ 14 &= (\mathbf{2})(\mathbf{7}) \\ \text{LCM} &\quad (\mathbf{2})^2(\mathbf{3})(\mathbf{5})(\mathbf{7}) \\ 420 &= (4)(3)(5)(7) \end{aligned}$$

An alternative to the prime factorization method for finding the least common multiple of two numbers is the repeated division process that was used in the previous chapter to reduce a fraction.

Find the LCM using the repeated division method

1) List the two numbers in the first row of the table.

2) On the outside of the table start with the smallest prime number that divides evenly into both of the numbers on that row and divide both the numbers in that row by that prime.

3) Continue this process of finding prime number divisors until no prime divisor divides evenly into the two numbers on a row of the table.

4) The LCM is the product of the prime number divisors.

Example 6 Find the LCM of 38 and 46 using the repeated division method.

Start by dividing 38 and 46 by the prime 2 which divides evenly into both numbers. Since no prime number divides evenly into both 19 and 23 stop the division process. The product (**2**)(19)(23) formed with the prime divisor **2** and last row 19 and 23 as factors when multiplied generates the LCM 874 as shown below.

$$\begin{array}{r|c|c}
\mathbf{2} & 38 & 46 \\ \hline
 & 19 & 23
\end{array}$$

2 divides evenly into both numbers

No prime divides into both 19 and 23

LCM is (**2**)(19)(23) = (38)(23) = 874

Example 7 Find the LCM of 36 and 40 using the repeated division method.

Start by dividing 36 and 40 by the prime number 2 which divides evenly into both numbers. Then divide 18 and 20 by the prime number 2. Since no prime number divides evenly into both 9 and 10 stop the division process. The product (**2**)(**2**)(9)(10) formed with the prime divisors **2** and **2** and last row 9 and 10 as factors when multiplied generates the LCM 360 as shown below.

$$\begin{array}{r|c|c}
\mathbf{2} & 36 & 40 \\ \hline
\mathbf{2} & 18 & 20 \\ \hline
 & 9 & 10
\end{array}$$

2 divides evenly into both numbers

2 divides evenly into both numbers

No prime divides into both 9 and 10

LCM is (**2**)(**2**)(9)(10) = (4)(90) = 360

Example 9 Find the LCM of 18 and 30 using the repeated division method.

Start by dividing 18 and 30 by the prime number 2 which divides evenly into both numbers. Then divide 9 and 15 by the prime number 3. Since no prime number divides evenly into both 3 and 5 stop the division process. The product (**2**)(**3**)(3)(5) formed with the prime divisors **2** and **3** and last row 3 and 5 as factors when multiplied generates the LCM 90 as shown below.

$$\begin{array}{r|c|c}
\mathbf{2} & 18 & 30 \\ \hline
\mathbf{3} & 9 & 15 \\ \hline
 & 3 & 5
\end{array}$$

2 divides evenly into both numbers

3 divides evenly into both numbers

No prime divides into both 3 and 5

LCM is (**2**)(**3**)(3)(5) = (9)(10) = 90

Exercises 3.1

1-15 Find the LCM of the following by listing the multiples of the largest number.

1.	6 and 8	2.	4 and 10	3.	8 and 12
4.	9 and 12	5.	5 and 10	6.	4 and 8
7.	10 and 12	8.	5 and 6	9.	4 and 9
10.	6 and 10	11.	4 and 18	12.	6 and 16
13.	2, 3 and 4	14.	5, 6 and 10	15.	4, 5 and 12

16-24 Find the LCM of the following **using the prime factorization method.**

16.	10 and 14	17.	10 and 18	18.	6 and 14
19.	8 and 20	20.	25 and 30	21.	18 and 22
22.	4, 5 and 6	23.	10, 15 and 18	24.	6, 9 and 15

25-30 Find the LCM of the following **using the repeated division method**.

25.	12 and 21	26.	24 and 60	27.	15 and 25
28.	30 and 42	29.	28 and 36	30.	22 and 26