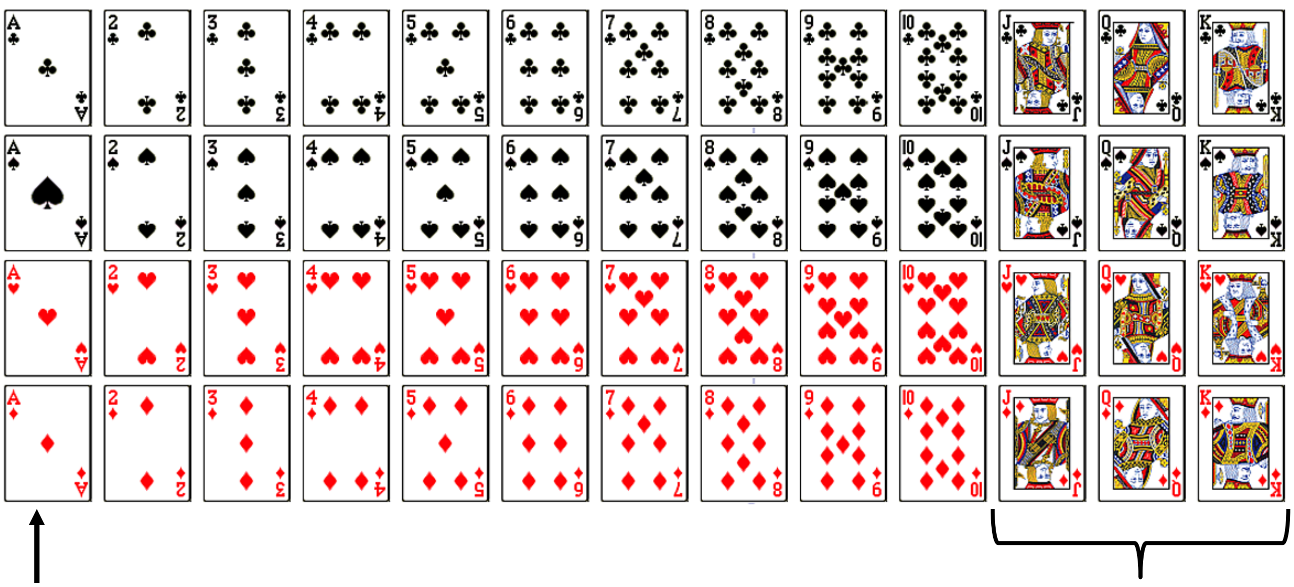
Probability

5.1 Overview

Standard Deck of Cards

- There are a total of 52 cards in a standard deck of cards. While most decks also come with two jokers, they are not used in most games of chance and are not counted in the 52 cards. We will not consider the jokers in our experiments.
- Half the cards are red and the other half are black (52 ÷ 2 = 26 red cards and 26 black cards)
- The deck is also equally divided into **four suits: hearts, diamonds, clubs and spades**. Each suit has 13 cards in it. (52 ÷ 4 = 13) The **hearts and diamonds are red**. The **clubs and spades are black**.
- There are **three face cards** in each suit**: king, queen & jack.** There is one ace in each suit. And the remaining 9 cards are numbered 2 through 10. The ace can be considered either the highest card (above the king) or the lowest card (as a 1). You will need to read each problem carefully to know if the ace is considered high or low.



Aces can be high (above the King) or low (like a one)

Face Cards are Jacks, Queens, and Kings

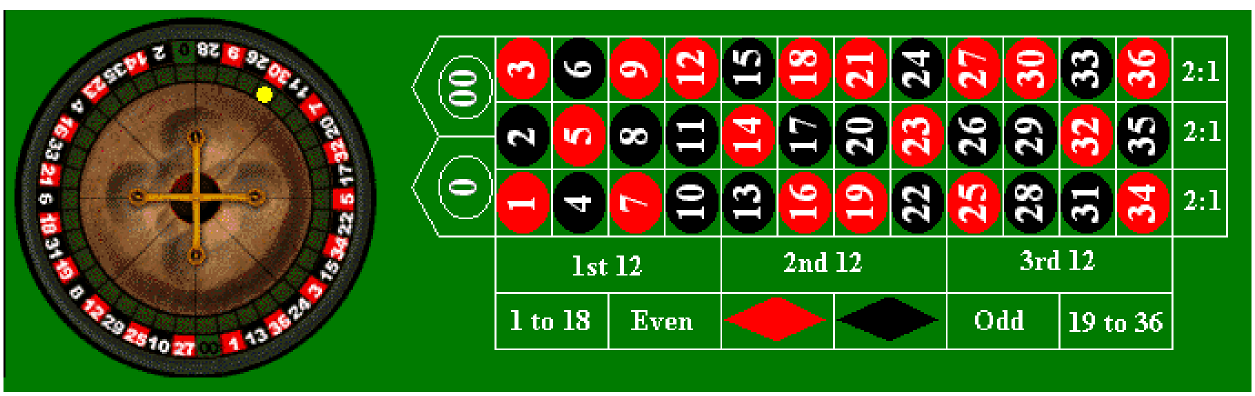
The American Roulette Wheel

American roulette will be another common example in this chapter. Roulette is played by spinning a wheel and dropping a small ball onto the wheel, the winner is determined by which space the ball finally ends up in.

You should learn the composition of the wheel.

- There are 38 spaces total
- There are 2 green spaces, numbered 0 and 00.
- The remaining 36 spaces are numbered 1 – 36.
- Half of the spaces numbered 1 – 36 are black, the other half are red. (36 ÷ 2 = 18 black spaces and 18 red)
- The 0 and 00 are always part of the sample space, but they do not count as low numbers (or even numbers) when placing a bet.

The Roulette Table looks like this:



To place a bet you put your chips/money on the board. To bet on a single number you place your chips on that number. To bet on two numbers, called a SPLIT, you place your chips on the line between the two numbers. You can only bet on two numbers that are neighbors. For example you can bet on the 1-2 split, but not 1 & 5 or 1 & 8. The other possible bets are:

- A three number Street
- A four number Square
- A five number Line
- A six number Line
- A group of 12 numbers, either a column or section
- The Low Numbers: 1 – 18 (don't include 0 or 00)
- The High Numbers: 19 – 39
- The Even Numbers: 2, 4, 6, … , 36 (don't include 0 or 00)
- The Odd Numbers: 1, 3, 5, … , 35
- The Red Numbers
- The Black Numbers

Probability Overview
How Do Casinos Make Money?

Have you ever walked into a casino before? It is an interesting experience. When you enter, you may notice a few things, depending on the casino:

- There are no clocks.
- The windows are tinted.
- The temperature is on the cool side (never hot).
- When you win while playing a machine it makes a fun and gratifying sound.
- They will bring you free drinks, but the drinks usually take a while to arrive.
- The layout of the casino is like a maze.

Many people think that the machines are rigged or that the casino cheats in order to make money. Casinos don't need to cheat to make money. They have probability on their side, and we will see that the Casinos get rich by exploiting a very small advantage a very large number of times.

In order to understand how Casinos make money, we will need to learn about various aspects of probability.

The first part of the Probability Module involves working with sets and tables. These are tools that will help us organize our data.

The second part focuses on Probability. We will look at the **Classical Approach to Probability** (also known as **Theoretic Probability**), which has its roots in gambling:

- **Probability** – this is a way of quantifying the likelihood that an event will occur.
- **Odds and Probability** – you hear about odds all the time in gambling. We'll learn what that means.
- **Counting Techniques** – classical probability involves counting possibilities. We'll learn various techniques and formulas to help us count possibilities quickly.

And we will look at the **Relative Frequency Approach to Probability** (also known as the **Experimental Approach**). In this approach, we run experiments and collect data in order to arrive at our probabilities. To work with this approach we will be studying:

- **Sets and Venn Diagrams** – we will use these as tools to organize our data.
- **Probability Distributions** – tables that show us the probability of different events
- **Two Way Frequency Tables** – Two way frequency tables will allow us to compare across two different categories. For example, we can use them to answer questions like "Are men more likely to support marijuana legalization than women are?"

We will see how the Classical Approach and the Relative Frequency Approach are related and give the same probabilities in the **Experimental Probability Lab**.

5.2 Working with Sets

Computing probabilities (one of our goals in this chapter) require working with sets. Sets are just groups of things, like a set of dishes or a set of sheets or the set of students in your Math 112 class. Objects in the same set typically have some shared characteristic like the pattern on your dishes or the fact that you are enrolled in the same math course. When working with numbers we have four basic operations: addition, subtraction, multiplication, and division. To work with sets we need to understand their operations: union, intersection, and complement.

Objectives

1. You will understand the language of sets
2. You will be able to find the **union** and **intersection** of two sets.
3. You will be able to find the **complement** of a set.
4. You will be able to tell if the two sets are **disjoint.**
5. You will be able to create and use **Venn Diagrams.**

First we need some vocabulary.

Universe: this is just what it sounds like – the universe. It is often denoted with a capitol letter U. The universe tells us what we are working with for each problem. We ONLY work with what is in our Universe. If our Universe is all the students enrolled in Math 112, there are no other students or people. In probability we often refer to the Universe as our **Sample Space**.

Sets: are groups of objects from our Universe. If our Universe is all the students enrolled in Math 112, then your section of Math 112 is a set. The universe is considered a set. For example our universe could be U = {1, 2, 3, 4, 5, 6}. Note that we denote sets with braces { }.

Elements: these are the objects in a set. If our Universe is all the students enrolled in Math 112, then you are one element in our Universe. If our universe is U = {1, 2, 3, 4, 5, 6}, then the number 4 is an element.

Subsets: these are the sets we can make using some (or all) of the elements in another set. If our set is all the students in enrolled in Dr. Conrad's Math 112 section, then a subset of that is the set of all the females in Mr. Conrad's Math 112 section. If our set is D = {1, 2, 3, 4, 5, 6}, then set A = {2, 4, 6} is a subset of set D.

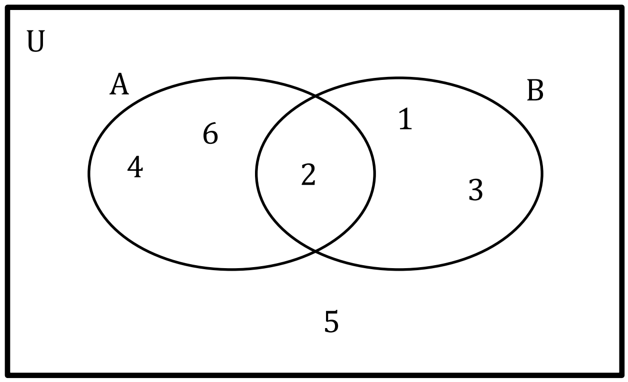
Disjoint: two sets are said to be disjoint if they do not have any elements in common. If our Universe is all the students enrolled in Math 112, then Mrs. Rhoads's section of Math 112 is disjoint from Mrs. Villatoro's section, because there are no students who are enrolled in both classes at the same time. If our universe is U = {1, 2, 3, 4, 5, 6}, then A = {2, 4, 6} and C = {1, 3} are disjoint.

Empty Set: the empty set is just that, a set with no elements. It is denoted either { } or Ø.

The Union of Two Sets

We can use a **Venn diagram** to show how these two sets relate to each other. The large rectangle contains our universe, all the possible elements. Each oval represents an individual set. They are typically overlapped because sets often share elements.

Let's start with U = {1, 2, 3, 4, 5, 6} and two sets: Let *A* = the set of even numbers = {2, 4, 6} and let *B* = the set of numbers less than four = {1, 2, 3}

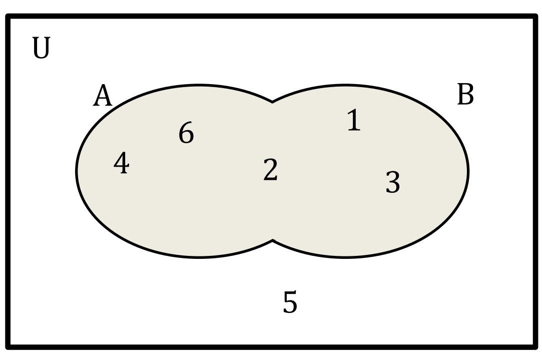


Because set A = {2, 4, 6} we draw a loop around those elements. Because set B = {1, 2, 3} we draw a loop around those elements. Note that we only wrote the number 2 down once. Since it is in both sets it needs to be inside both loops. The number five is left outside of both sets, because it is in neither set. But 5 is part of our Universe so we must include it in our Venn diagram.

If we want to add these two sets together we would take their **union.** We use this symbol $\cup$. We place the union symbol between the two sets: $A \cup B$

$$A \cup B = \{1, 2, 3, 4, 6\}$$

Note that $A \cup B$ contains any number that is even (in set A) OR less than 4 (in set B). On the Venn diagram the union contains all the elements that are inside either loop.

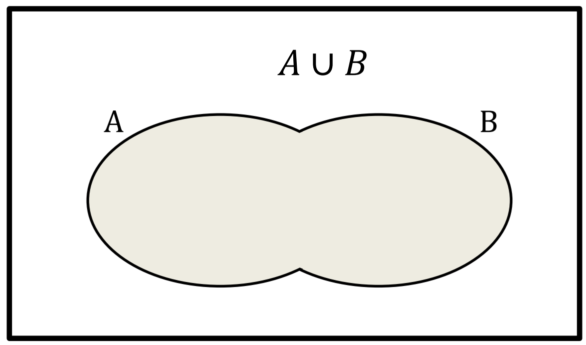


The **union** of sets A and B is a new set that contains all the elements of set A mixed with all the elements of set B. The union is denoted: $A \cup B$ and consists of all elements that are either in set A OR in set B OR in both.

The phrase "A or B" should be thought of as the union: $A \cup B$. To be a member you must either be in A or in B (or both). This is the same way we typically use the word "or" outside of a math class. For example to join the Cal Tech Credit union you must be an employee of Cal Tech **or** JPL. Paul works for JPL so he can join the credit union. Kenji works at Cal Tech so he can also join. Mary works part time at both locations, she can also join the credit union.

In general, the Union of two sets A and B is:

The Union of two sets is a new set that contains all the elements of the original two sets.



We will use this idea many times answering our probability questions.

Example 1: Let U = {a standard deck of cards}, H = {all the hearts}, and F = {all the face cards}. What is $H \cup F$?

> To take the union of two sets means to combine them into one new set that contains all the elements that are in either set. So $H \cup F$ is all the cards that are either hearts OR face cards (or both, like the king of hearts).
>
> $H \cup F$ = {all the hearts; the King, Queen, Jack of clubs; the King, Queen, Jack of spades; the King, Queen, Jack of diamonds}

Example 2: Let IA = {the students enrolled in Math 112} and F = {female students at SCC}. What is $IA \cup F$ = ?

> The union of these two sets, $IA \cup F$, would contain any student who is either enrolled in Math 112 or is a female student at SCC. The males enrolled in Math 112 would be part of this set, because they are elements of set IA. Women enrolled in Math 20: Calculus would also be in the union because they are female students at SCC.

The Intersection of Two Sets

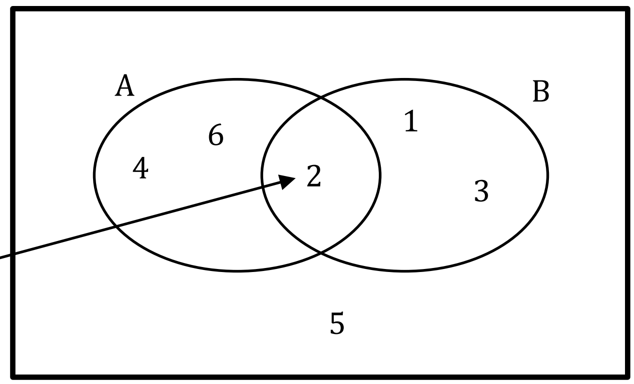
Let's go back to U = {1, 2, 3, 4, 5, 6}, A = {2, 4, 6} and B = {1, 2, 3}. The **intersection** of two sets is denoted with the symbol: $\cap$. A intersect B, $A \cap B$, is the set containing only those elements that are in BOTH A and B. $A \cap B$ = {2} because 2 is the only element that appears in both sets.

The **intersection** of two sets is a set that consists of only the elements that are both in A **<u>and</u>** B at the same time. This set is denoted: $A \cap B$.

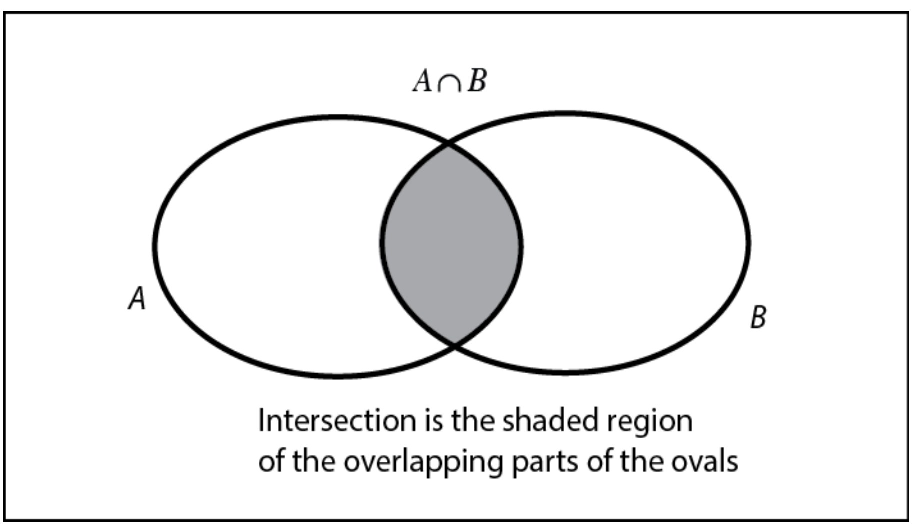
The phrase "A and B" is a condition that means your element needs to be in both A and B simultaneously.

If we look at our Venn diagram, we see that $A \cap B$ is comprised of elements in the overlap of the two sets *A* and *B*.

$A \cap B$ is the region where the two sets overlap.



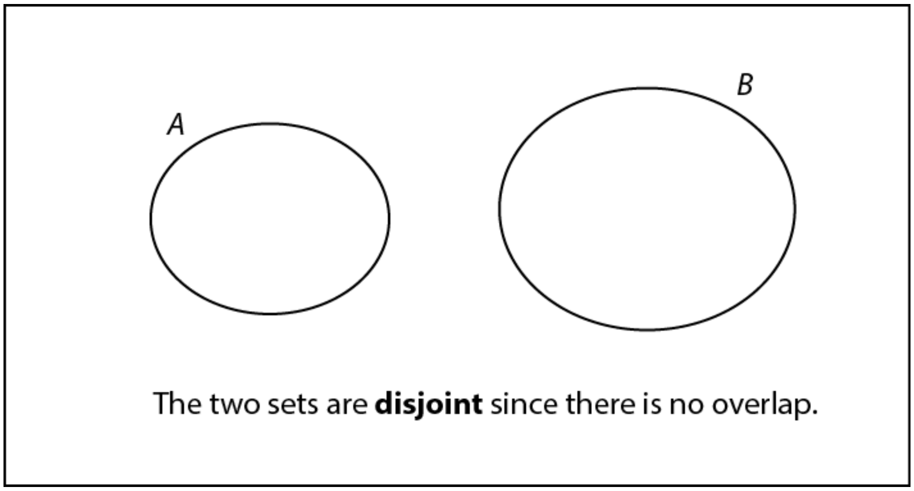
In general, the intersection of two sets *A* and *B* is:



Example 3: Let U = {a standard deck of cards}, H = {all the hearts}, and F = {all the face cards}. What is $F \cap H$?

> The intersection of these two sets would be only those cards that are in both A and B. $F \cap H$ = {King of hearts, Queen of hearts, Jack of hearts}.

If two sets don't have any common (or shared) elements, we say the sets are **disjoint**.



The two sets are **disjoint** since there is no overlap.

When a set has nothing in it we say it is **empty**. The set with nothing in it is called the **empty set** and has two standard notations: { } and Æ.

Thus, A and B are disjoint if $A \cap B = \{\,\}$

Example 4: Let U = {a standard deck of cards}, H = {all the hearts}, and F = {all the face cards}. Are H and F disjoint?

> No, they are not disjoint because as we saw in the previous example that there are three cards that are in both sets.

Example 5: Let U = {a standard deck of cards}, H = {all the hearts}, and C = {all the clubs}. Are H and C disjoint?

> Yes, they are disjoint because there is NO single card in the deck that has both a red heart and a black club on it.

The Complement of a Set

Let's continue with U = {1, 2, 3, 4, 5, 6}, A = {2, 4, 6} and B = {1, 2, 3}. The complement of A is written: $\overline{A}$ or A' and is the set containing all the elements from our universe that are NOT inside set A. Since set A is all the even numbers in our universe, then A complement must be all the odd numbers: $A' = \{1, 3, 5\}$.

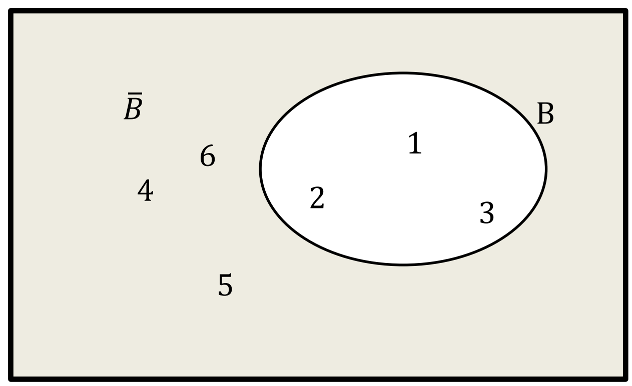
> The **complement** of a set is a set that that consists of all elements in the sample space that are NOT in A. This set is denoted: $\overline{A}$ or A' .

Using our Venn diagram to find the complement of set B we compare set B to our Sample Space or Universe,

Sample Space U = {1,2,3,4,5,6}

$$B = \{1,2,3\}$$

$$\overline{B} = B' = \{4,5,6\}$$



Example 6: Let U = {a standard deck of cards} and F = {all the face cards}. Are F and $\overline{F}$ disjoint?

Since set F is all the Face Cards, $\overline{F}$ is the set of all the cards that are NOT face cards.

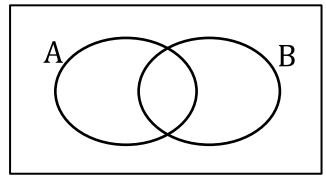
F = {King, Queen, Jack from each suit}

$\overline{F}$ = {Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10 from each suit}

Yes, these two sets are disjoint. In set notation: $F \cap \overline{F} = \varnothing$

Note: This is true for every set and its complement! Any set is disjoint from its complement.

Classwork 5.2



Draw some Venn Diagrams like this.

Then color in the indicated region on each Venn Diagram.

1. $A \cup B$
2. $A \cap B$
3. $A \cup \bar{B}$
4. $A \cup B'$
5. $\bar{A} \cap B$
6. $A' \cap B'$

Suppose our sample space is S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {2, 3, 5, 7} and E = {0, 2, 4, 6, 8, 10}

7. Draw a Venn diagram to represent this situation. Be sure to include all the elements of our sample space.
8. $P \cup E =$
9. $E \cup P =$
10. What do you notice about your answers to the last two problems?
11. $P \cap E =$
12. E¢=
13. $\bar{E} =$
14. $\bar{P} =$
15. $E \cup P' =$
16. $E \cap \varnothing =$
17. $P \cup P' =$
18. $P \cap \bar{P} =$
19. Create a new set A, from our set S, that is disjoint with set P.
20. Create a new set B, from our set S, that is disjoint with set E.

Suppose our universe, U, is a standard deck of cards. Let set H = {the hearts}, F = {face cards}, R = {red cards}, and B = {black cards}. Describe the elements (cards) for each of the following:

21. $F \cap B =$
22. $H \cap B =$
23. Are H and B disjoint? Explain your answer.
24. Are F and H disjoint? Explain your answer.
25. $\bar{B} =$
26. $H \cap B' =$
27. $B \cup F =$
28. $B \cup R =$
29. $H \cap \bar{F} =$

Exercises 5.2

1. Write definitions of **sample space, union, intersection, disjoint** and **complement** in your own words. Give examples.

Suppose the sample space is {1, 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10}.

Let A = the set of even numbers in the sample space.
Let B = the set of numbers that are **prime** (a number is prime if the only numbers that divide into it without a remainder are 1 and itself. For example, 17 is prime because only 1 and 17 divide into it without a remainder. 14 is not prime, because 2 divides into 14 without a remainder. So does 7.)

2. Draw a Venn diagram to represent this situation. Be sure to include all the elements of our sample space.
3. $A \cap B =$
4. $A \cup B =$
5. $\bar{A} =$
6. $\bar{B} =$
7. Are B and $\overline{B}$ disjoint? Explain.
8. Are A and B disjoint? Explain.
9. Is a set and its complement always disjoint? Why or why not?
10. Create a new set C so that C is disjoint from A.
11. Create a new set D so that the intersection of B and D is empty.
12. Create a new set E that is NOT disjoint with B.

Let U = {all the people living in the Bay Area}, A = {all the 49er fans}, D ={all the Raider fans}. Describe which teams the people in each of the following sets like or don't like. Remember the union relates to "or" and the intersection to "and"

13. $\bar{A} =$
14. D¢=
15. $A \cap D =$
16. $A \cup D =$
17. $A \cap A' =$
18. $\bar{D} \cup D =$
19. $A' \cap D =$
20. $\bar{A} \cup D =$

5.3 Working with Sets and Tables

Using Tables to describe Sets

Most of the time we will be most interested in how many elements are in each set. We can use tables to easily represent this. For example here is a table showing how many cards belong in each category if we are looking at face cards and clubs.

	Club	Not a Club	Totals
Face Card	3	9	12
Not a Face Card	10	30	40
Totals	13	39	52

Note that if you add down the total column on the right you get 52 cards, to whole deck. This is also true if you add across the bottom row. The rows separate our deck in Face Cards and non-Face Cards. The columns separate the Clubs from the other cards.

Using the table we can answer lots of questions.

Example 1: Let U = {our deck of cards}, F = {the face cards}, and C = {the clubs}. How many cards are in each of the following categories?

- How many clubs are there? From the table above we get: 3 + 10 = 13
- How many cards are not clubs? From the table above we get: 9 + 30 = 39
- How many cards are clubs and face cards? 3 (recall: this is the intersection, $C \cap F$)
- How many cards are clubs or face cards? We need to count any card that is either a club OR a face card. 3 + 10 + 9 = 22 (recall: this is the union of the sets, $C \cup F$)
- How many cards are neither clubs nor face cards? 30

Example 2: Use the table to answer the following questions.

	Democrat	Republican	Totals
Age ≤ 55	45	29	74
Age > 55	28	80	108
Totals	73	109	182

- How many Republicans are there? ___________
- How many Democrats? __________
- How many people over age 55? __________
- How many people are 55 or younger? ____________
- How many people are Democrats and over 55? ____________
- How many are Democrats or over 55? ___________
- How many are neither Republicans nor over 55? ______

Classwork 5.3a

Use this table to answer the following questions. Let D = {diamonds} and F = {face cards}

	Diamond	Not a Diamond	Totals
Face Card	3	9	12
Not a Face Card	10	30	40
Totals	13	39	52

1. How many are diamonds?
2. How many are not face cards?
3. How many are diamonds and face cards?
4. How many are in the complement of D?
5. How many are in $D \cap F$?
6. How many are in $D' \cap F$?
7. How many are in $D \cap F'$?
8. How many are in $\bar{D} \cap \bar{F}$?
9. How many are in $F \cup D$?
10. How many are in $F \cup \bar{D}$?
11. How many are in $F' \cup D$?
12. How many are in $\bar{F} \cup \bar{D}$?

Use this table to answer the following questions. Let M = {students taking Math} and E = {students taking English}

	Taking Math	Not taking Math	Totals
Taking English	43	19	62
Not taking Eng.	10	30	40
Totals	53	49	102

13. How many are taking both?
14. How many are taking neither?
15. How many are taking English, but are not taking Math?
16. How many students are not taking Math?
17. How many are taking Math, but are not taking English?
18. How many are taking either Math or English?
19. How many are in $M \cup E$?
20. How many are in $M \cap E$?
21. How many are in $M \cup \bar{E}$?
22. How many are in $\bar{M} \cup E$?
23. How many are in $M \cap E'$?
24. How many are in $\bar{M} \cap E$?
25. Of the students taking English, how many are taking Math?
26. Of the students taking Math, how many are not taking English?
27. Of the students not taking Math, how many are taking English?

Exercises 5.3a

Use this table to answer the following questions. Let D = {Democrats} and V = {Voted in the last election}

	Democrat	Republican	Totals
Voted	85	45	130
Did not Vote	37	12	49
Totals	122	57	179

1. How many people did not vote?
2. How many people are Republicans?
3. How many people are Democrats who did not vote?
4. How many people did vote, but are not Democrats?
5. Of the people who voted, how many are Democrats?
6. How many people are Republicans who did vote?
7. Of the Republicans, how many did not vote?
8. How many are in $D \cup V$?
9. How many are in $D \cup \bar{V}$?
10. How many are in $D \cap \bar{V}$?
11. How many are in $D' \cap \bar{V}$

Use this table to answer the following questions. Let Y = {Age < 30} and M = {support marijuana legalization}

	Support	Don't Support	Totals
Age < 30	120	75	195
Age ≥ 30	80	92	172
Totals	200	167	367

12. How many people do support marijuana legalization?
13. Of the people under 30, how many support legalization?
14. How many people who are 30 or older do not support legalization?
15. How many people are in $\bar{Y} \cap \bar{M}$?
16. How many people are in $Y \cap M$?
17. How many people are in $\bar{Y} \cap M$?
18. How many people are in $\bar{Y} \cup \bar{M}$?
19. How many people are in $Y \cup M$?
20. How many people are under 30 or support legalization?
21. How many people are under 30 and support legalization?
22. How many people are in $\bar{Y}$ =
23. How many people are in the complement of M?
24. Are Y and M disjoint? Explain.
25. Look back at #13,16, and 21. Is it a coincidence that they all have the same answer?
26. What do you notice about #14 and 15?
27. What do you notice about # 19 and 20?

Making Tables

Example 3: Use the following information to create a table representing the results of our survey. In our survey we asked if people spoke Spanish or Chinese. A total of 173 people were surveyed. 45 people speak both languages. A total of 65 people speak Chinese and 90 do not speak either language.

First we start by making our table.

	Spanish	Not Spanish	Totals
Chinese			
Not Chinese			
Totals			

Then we start filling in what we know. "45 people speak both languages," so we know 45 goes in the upper left box. And "A total of 173 people were surveyed," tells us that 173 goes in the lower right box.

	Spanish	Not Spanish	Totals
Chinese	45		
Not Chinese			
Totals			173

We know there is a total of 65 people who speak Chinese, so we put 65 in the upper right.

	Spanish	Not Spanish	Totals
Chinese	45		**65**
Not Chinese			
Totals			173

We need 20 more people in the top row because we solved: $45 + x = 65$.

	Spanish	Not Spanish	Totals
Chinese	45	**20**	65
Not Chinese			
Totals			173

90 people do not speak either language, so we need 90 below the 20. This is where "Not Spanish" overlaps with "Not Chinese".

	Spanish	Not Spanish	Totals
Chinese	45	20	65
Not Chinese		**90**	
Totals			173

Adding down the "Not Spanish" column gives us a total of 110. We can calculate the missing number in the column on the far right by solving 65 + x = 173, so x = 108.

	Spanish	Not Spanish	Totals
Chinese	45	20	65
Not Chinese		90	**108**
Totals		**110**	173

We can complete the "Not Chinese" row by solving: x + 90 = 108, so x = 18. And that will let us get the total in the "Spanish" column.

	Spanish	Not Spanish	Totals
Chinese	45	20	65
Not Chinese	**18**	90	108
Totals	**63**	110	173

Classwork 5.3b

Use the given information to make a 4 by 4 table that displays all the information, including the totals.

1. We surveyed 150 people. We asked them two questions: Do you support Trump? Are you more than 25 years old? Here are the results:
    a. A total of 95 people supported Trump, 55 did not.
    b. A total of 50 people are over 25 years old, the rest are not.
    c. 35 people who are over 25 supported Trump.
    d. 15 people who are over 25 did NOT support Trump.
    e. Of the people who are 25 or less, 60 of them do support Trump.

2. We went back and asked the same 150 people two more questions: Are you married? Are you a Democrat? Here are the results:
    a. 65 are Democrats
    b. 25 of the Democrats are married, the rest are not
    c. 85 are not Democrats
    d. 90 people are married
    e. 20 people are not married and are not Democrats

Exercises 5.3b

Use the given information to make a 4 by 4 table that displays all the information, including the totals. There is more than one correct table, depending on how you label your rows and columns.

1. We surveyed 200 people. We asked them: Are you 21 or older? Have you ever consumed alcohol? Here are the results:
   a. 65 people are 21 or older and have consumed alcohol
   b. 50 people are under 21 and have consumed alcohol
   c. 30 of the people 21 or older have not consumed alcohol
   d. 85 people said they had not consumed alcohol

2. We surveyed 355 people. We asked them: Did you vote in the last presidential election? Are you a registered voter? Here are the results:
   a. Only 130 of the registered voters actually voted
   b. There was a total of 230 registered voters in our sample
   c. 125 people were not registered and did not vote
   d. If you are not registered, you cannot vote

3. We surveyed 442 people. We asked: Do you support the Raiders? Do you support the 49ers? Here are the results:
   a. 200 people did not support either team.
   b. 87 people support both teams.
   c. 92 support the Raiders, but not the 49ers
   d. A total of 263 do not support the Raiders.

4. We surveyed 239 travelers at SFO. We asked if they had been to Tibet or to Nepal. Here are the results:
   a. 32 people had been to both
   b. 100 had been to neither
   c. A total of 150 had not been to Nepal
   d. 57 had been to Nepal, but not to Tibet

5. We surveyed 132 students at SCC. We asked if they were taking either Math or Spanish.
   a. 17 were taking both classes
   b. 65 were taking neither class
   c. Only 29 were taking Math

5.4 Working with Venn Diagrams and Tables

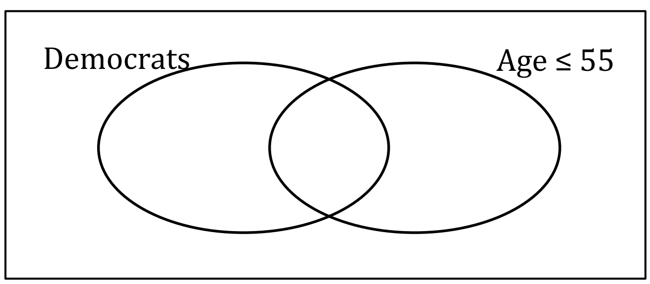
We can use Venn Diagrams to organize data the same way we used tables. Instead of a table like this:

	Democrat	Republican	Totals
Age ≤ 55	45	29	74
Age > 55	28	80	108
Totals	73	109	182

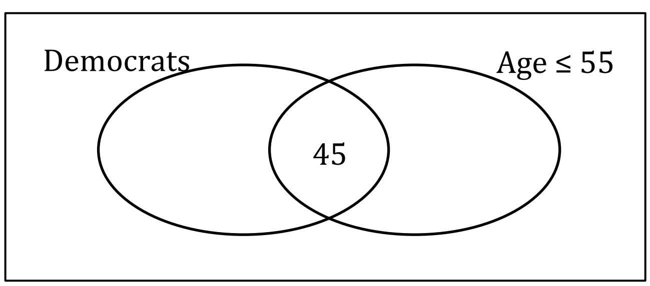
We can organize the same information using a Venn Diagram. We only need two sets, one for political party and one set for age. The person making the Venn Diagram needs to decide to focus on either Democrats or Republicans and either people over 55 or the younger folks. If there are only two options and you make one set for Democrats then the Republicans are outside the set of Democrats.

Example 1: Use Democrats and people 55 or younger as your two sets and make a Venn Diagram that represents the data in the table above.

We start by drawing our Venn Diagram and labeling our sets.

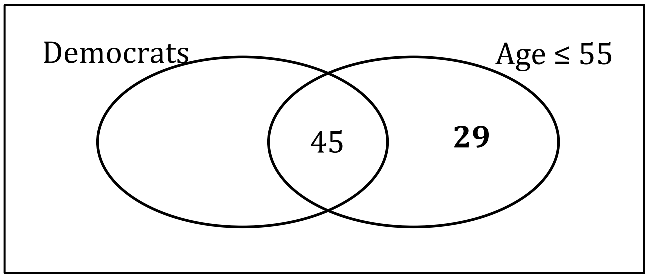


From the table we can see that 45 people are Democrats and are 55 or under, so they go in the intersection of our two sets.

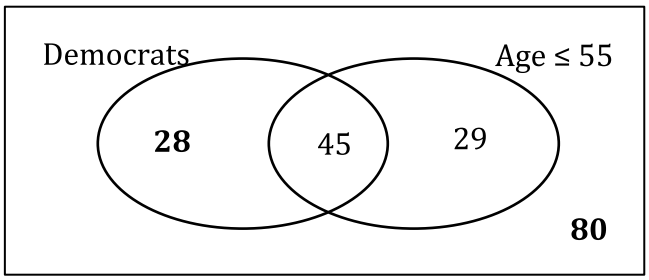


Working across the top row of the table we see there are 29 Republicans who are 55 or under. They have to be inside the set on the right, but not inside the Democrat set.

Like this:

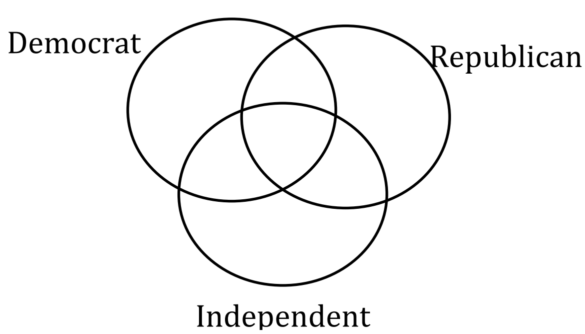


The table also tells us that there are 28 Democrats who are over 55. They go in the left portion of the Democrat set. The last 80 people are older Republicans. They go outside both sets.



NOTE: When making Venn Diagrams there is always more than one possible answer, depending on which sets you choose to focus on. The location of the four values may change, but any correct answer should have the same four numbers. You want to choose your two sets so that each one represents one category or question. We do not need a set for "over 55 years old" and another set for "55 or younger" because they are complements of each other. We assume the same for Democrats and Republicans, we assume everyone outside the Democrat set is a Republican.

Often there are more than just two possibilities. For example: Democrat, Republican, and Independent. In those situations we need more sets.



In this book we will focus on Venn Diagrams that can be completed with only two sets.

Classwork 5.4

For each problem, create a Venn Diagram that represents the data given.

1. Redo the example above using Republicans and Age > 55 as your two sets.

2. What do you notice about your answer to #1 and the example above?

3. Use Democrat and Voted as your two sets.

	Democrat	Republican	Totals
Voted	85	45	130
Did not Vote	37	12	49
Totals	122	57	179

4. Use Diamond and Face Card as your two sets.

	Diamond	Not a Diamond	Totals
Face Card	3	9	12
Not a Face Card	10	30	40
Totals	13	39	52

5. We surveyed 160 people. We asked them two questions: Do you support Trump? Are you more than 25 years old? Here are the results:
   a. A total of 105 people supported Trump, 55 did not.
   b. A total of 50 people are over 25 years old, the rest are not.
   c. 35 people who are over 25 supported Trump.
   d. 15 people who are over 25 did NOT support Trump.

6. We surveyed 239 travelers at SFO. We asked if they had been to Tibet or to Nepal. Here are the results:
   a. 32 people had been to both
   b. 100 had been to neither
   c. A total of 150 had not been to Nepal
   d. 57 had been to Nepal, but not to Tibet

Exercises 5.4

Create a Venn Diagram that represents the data. Remember that there is more than one possible correct answer depending on which characteristics you focus on.

1.

	Taking Math	Not taking Math	Totals
Taking English	43	19	62
Not taking Eng.	10	30	40
Totals	53	49	102

2.

	Support	Don't Support	Totals
Age < 30	120	75	195
Age ≥ 30	80	92	172
Totals	200	167	367

3. We went back and asked 150 people two questions: Are you married? Are you a Democrat? Here are the results:
    a. 65 are Democrats
    b. 25 of the Democrats are married, the rest are not
    c. 85 are not Democrats
    d. 90 people are married
    e. 20 people are not married and are not Democrats

4. We surveyed 442 people. We asked: Do you support the Raiders? Do you support the 49ers? Here are the results:
    a. 200 people did not support either team.
    b. 87 people support both teams.
    c. 92 support the Raiders, but not the 49ers
    d. A total of 263 do not support the Raiders.

5. We surveyed 132 students at SCC. We asked if they were taking either Math or Spanish.
    a. 17 were taking both classes
    b. 65 were taking neither class
    c. Only 29 were taking Math

5.5 Probability: The Mathematics of Uncertainty

Introduction

We live in a world filled with uncertainty. We often wish that there were more certainty in life. It would be nice to know which lottery ticket is the winner. Or that coming to class every day would guarantee a passing grade. Unfortunately, very little in life is certain.
Probability is a branch of mathematics that was developed to help us understand how likely something is (or isn't). The roots of probability are securely based in the art of gambling. Today probability plays a role in many aspects of our lives.

- What is the chance your child is left handed? Or has blue eyes?
- How likely are you to be in a car accident?
- How much you will be charged for car insurance?
- Are you more likely to be hurt by a terrorist or stuck by lightning?
- Are you safer in a car or in an airplane?
- How likely are you to win the lottery?

All these questions can be answered using probability.

Objectives

1. You will be familiar with the terms experiment, outcome, event, simple event and sample space.
2. You will understand the definition of probability and be able to do calculations.
3. You will be able to calculate probabilities involving a deck of cards and an American Roulette wheel.

Definitions and Terminology

An **experiment** is a situation involving chance that results in some sort of **outcome**. An experiment can be almost anything. Many of our experiments will involve rolling dice, flipping coins, and picking cards from a standard deck of cards.

An **outcome** is a single observable result of an experiment. We use both terms interchangeably. If someone is hiding a piece of candy in one of their hands, and we are choosing which hand we think it is in, are only two possibilities: it is in their right hand or left hand. Each single result is considered an outcome for that experiment.

The set or group of all outcomes for an experiment is called the **sample space**. Traditionally we use the letter S to represent our sample space, and we list all the outcomes inside braces { } because they are sets. Recalling our work with sets, the Sample Space is like the Universe.

Definitions and Terminology, continued

An **event** is one or more outcomes of an experiment. We usually use capital letters like E and F to represent events. Events are sets, therefore we list the outcomes included in that event inside braces { }. An event containing only one outcome is called a **simple event**. When an event consists of more than one outcome we call it a **compound event**.

Example 1:

We roll a six-sided die. The act of rolling the die is the experiment. The outcomes are getting a 1, getting a 2, getting a 3, getting a 4, getting a 5, and getting a 6. The **sample space** is S = {1, 2, 3, 4, 5, 6}.

There are many possible events including:

- Getting a 5: F = {5} is a simple event
- Getting an even number: E = {2, 4, 6} is a compound event
- Getting an odd number: O = {1, 3, 5}
- Getting a 1, 2, or 3: A = {1, 2, 3}
- Getting a 7: an impossible event because 7 is not in our sample space, so we write { } or Ø for an empty set.
- Getting a number that is less than 8: a certain event because all the numbers in our sample space are less than 8
- … and many more …

Note: The “names” given to these events: F, E, O, and A are arbitrary and can be chosen or changed by whoever is doing the experiment.

The **probability** of an event occurring is the likelihood of an event occurring measured as a number between 0 and 1. When each outcome is equally likely:

$$\text{Probability of an event occurring} = \frac{\text{number of ways it can happen}}{\text{total number of outcomes}}$$

$$0 \leq \text{Probability} \leq 1$$

If an event is guaranteed to occur, its probability is 1. If an event is impossible, its probability is 0. Since fractions can be converted to percentages, often people think of a probability as being between 0% and 100% (or between 0 and 1). Probabilities are NEVER negative and are never larger than one.

Before starting any probability problems it is very important that we understand our experiment and our sample space. Our sample space tells us how many outcomes there are in total, and our events are always subsets of the sample space. We will be focusing on experiments where each outcome is equally likely.

<u>Example 2</u>:

Using our die rolling experiment from example 1, calculate the following probabilities. Because our sample space, S = {1, 2, 3, 4, 5, 6}, has six equally likely outcomes in it, therefore our denominator will always be 6.

- The probability of getting a 5. The probability is $\frac{1}{6}$ because the 5 only appears once out of the six possible numbers.
- The probability of getting an even number. Because this is a compound event {2, 4, 6} we have to include all three possibilities. The probability is $\frac{3}{6} = \frac{1}{2}$
- The probability of getting an odd number. The probability is also $\frac{3}{6} = \frac{1}{2}$ because there are also three outcomes in this event {1, 3, 5}
- The probability of getting a 7. This is an impossible event because 7 is not in our sample space. This means the probability is ZERO.
- The probability of getting a number that is less than 8. This is a certain event because all the numbers in our sample space are less than 8. The probability is $\frac{6}{6} = 1$

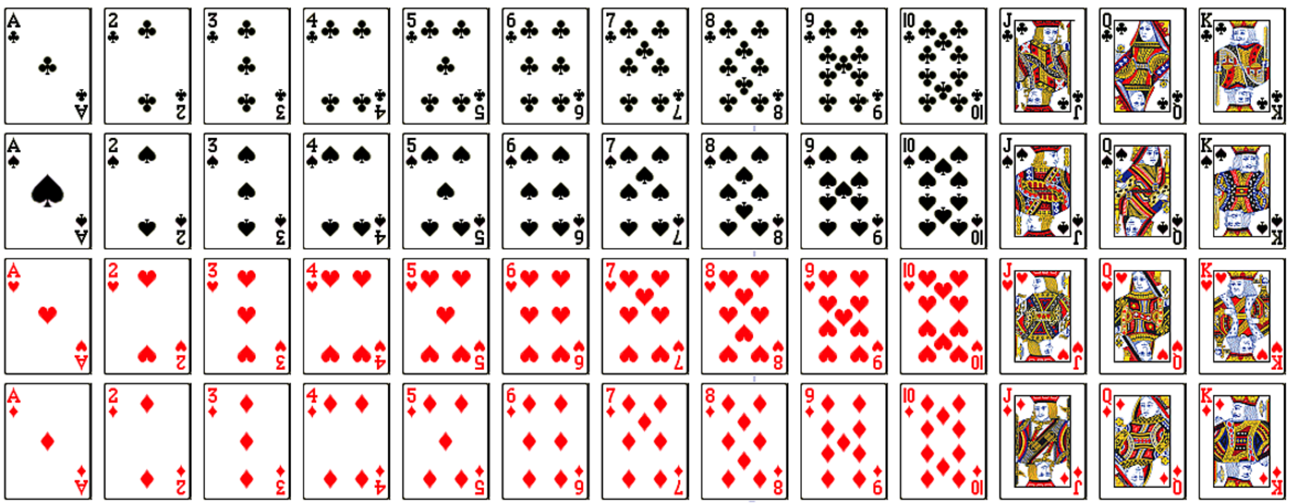
If you were to stop just about anyone on the street and ask them "what's the probability that when I flip this coin it lands on heads?" they would probably say 50 – 50 or 50% or maybe $\frac{1}{2}$. And as long as you were holding a fair coin they would be correct. The chance (or probability) that a coin lands on heads is $\frac{1}{2}$. That does NOT mean that if you flip a coin twice you will get one head and one tail! The **Law of Large Numbers** tells us that the more times you flip a coin, the closer you will get to having it land on heads half the time and tails the other half. But there is no guarantee that even with 1,000,000 flips it will land on head exactly 500,000 time and 500,000 times on tails.

In experimental probabilities (or relative frequencies) we have to run the experiment many times and collect lots of data. We will do a lab where we do many experiments. Most of your homework, on the other hand, will involve using classical (or theoretic) probabilities.

<u>Example 3</u>:

Our experiment is picking a single card from a standard deck of cards. Aces are low. (See the beginning of this chapter for more information about a deck of cards.)

- How big is the sample space? 52, because our deck contains 52 cards.
- What is the probability the card picked is the king of hearts? P(king of hearts) = $\frac{1}{52}$
- What is the probability the card picked is the king of spades? P(king of spades) = $\frac{1}{52}$
- What is the probability the card is a king? P(king from any suit) = $\frac{4}{52} = \frac{1}{13}$
- What is the probability the card is above an eight? P(above an eight) = $\frac{20}{52} = \frac{5}{13}$

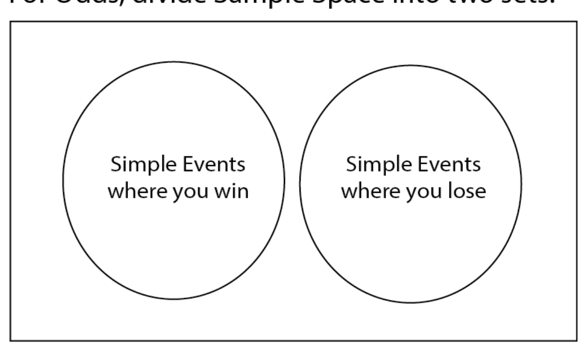


ODDS

We've been working with probabilities expressed as decimals or fractions. When you go to a casino or a race track, you will hear people talk about **odds** like 1 to 3. Odds and probability are closely related.

When we calculate a probability, we are comparing the outcomes we are interested in to the entire sample space. When we talk about the odds of an outcome or event happening we separate the sample space into two disjoint sets: the outcome(s) we want (the ones where we win) compared to all the other outcomes.

For Odds, divide Sample Space into two sets:



To calculate the odds, we look at the **ratio** of the number of simple events where we win compared to the number of simple events where we lose. This can be written as:

# of winning simple events **to** # of losing simple events
or
# of winning simple events **:** # of losing simple events

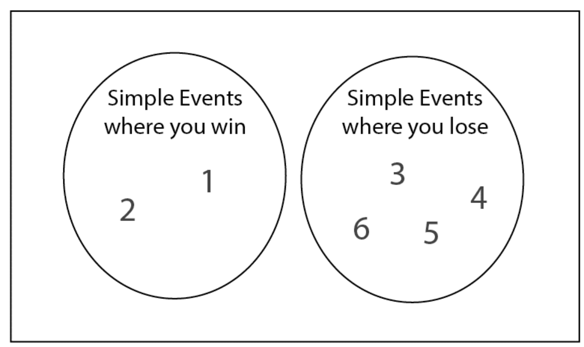
There is a direct link between Odds and the complement of a set. To compute the odds you compare how many objects are in your set to how many are in the complement of your set. You can simplify the ratio, just like you can simplify fractions, 2 to 4 can be simplified to 1 to 2. Similarly, 18:2 can be simplified to 9:1

Example 4: When rolling a six sided die, what are the odds of getting a number less than 3?

We take our sample space = {1, 2, 3, 4, 5, 6} and break it up into two disjoint sets:

1. the simple events where we win (getting a number less than 3) which consists of {1, 2}
2. the simple events where we lose (getting a number that is NOT less than 3) which consists of {3, 4, 5, 6}

Rolling a die and winning with a number less than 3



ODDS = 2 to 4
which reduces to 1 to 2

Example 5: If our experiment is picking one card from a standard deck what is the probability you get a face card AND what are the odds you get a face card?

Since there are 52 cards total and only 12 face cards the **probability is** $\frac{12}{52}$

To calculate the odds we need to count how many cards are not face cards (the complement). There are 52 total and 12 face cards, so there must be 52 – 12 = 40 cards that are not face cards. Therefore the **odds your card has a face on it are 12 : 40.**

Classwork 5.5

Using a standard deck of cards & picking just one card write both the ***probability and the odds*** for each event. Consider Aces as high cards. Review the deck of cards at the beginning of this chapter if needed. Give probabilities as fractions, un-simplified.

1. Picking a Heart
2. Picking a card that is NOT a Heart
3. What do you notice about # 1 and 2?
4. Picking a face card
5. Picking a red face card
6. Picking a card greater than 9
7. Picking a card greater than or equal to 9
8. Picking a card less than

Using our American Roulette wheel write both the ***probability and the odds*** for each event. (see the beginning of the chapter for more information about Roulette.) Remember that 0 and 00 are not counted as "low" nor "even" nor "high". They are always included in the sample space. Give probabilities as fractions, un-simplified.

9. The ball lands on a green space
10. The ball lands on a black space
11. The ball lands on red or green space
12. The ball lands on a number that is at least 7
13. The ball lands on a number that is at most 7
14. The ball lands on a number that is no more than 7
15. The ball lands on a number that is no less than 7
16. The ball lands on a number that is more than 40

From a bag that contains 5 red tokens, 6 green tokens, and 2 blue tokens, someone will reach in and pick one at random, without looking. Write probabilities as fractions, un-simplified, and as percentages rounded to the nearest tenth of a percent.

17. What is the probability you get a green token?
18. What are the odds you get a green token?
19. What are the odds you do not get a green token?
20. What percentage of the time will you get a red or a green token?
21. What is the probability you get red and a green token?
22. If R ={the red tokens} what is the probability your token is from $\overline{R}$?
23. What are the odds you get a token from $\overline{R}$?
24. What percentage of the time will you get a token from $\overline{R}$?

Exercises 5.5

Using a standard deck of cards & picking just one card write both the ***probability and the odds*** for each event. Consider Aces as high cards. Review the deck of cards at the beginning of this chapter if needed. Give probabilities as fractions, un-simplified.

1. Picking a Jack
2. Picking a Club
3. Picking a Jack and a Club
4. Picking a Jack OR a Club
5. Picking a card that is less than 5
6. Picking a card that is less than or equal to 5
7. Picking a card that is not more than 5

Using our American Roulette wheel write both the ***probability and the odds*** for each event. (see the beginning of the chapter for more information about Roulette.) Remember that 0 and 00 are not "low". Give probabilities as fractions, un-simplified. Don't count 0 or 00 as low.

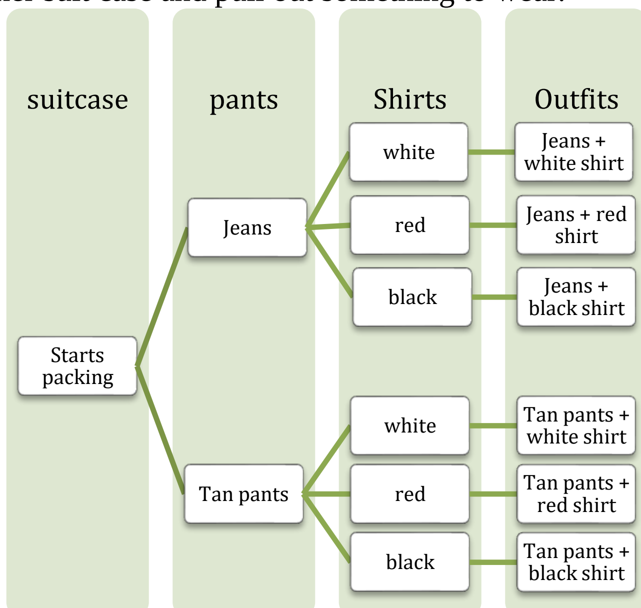
8. The ball lands on the number 13
9. The ball lands on a green space
10. The ball lands on a green space or the number 13
11. The ball lands on a number that is not more than 6
12. The ball lands on a number that is less than 6

From a bag that contains 8 red tokens, 7 green tokens, and 4 blue tokens, someone will reach in and pick one at random, without looking. Write probabilities as fractions, un-simplified, and as percentages rounded to the nearest tenth of a percent. Let R = the set of red tokens, G = the set of green tokens, and B = the set of blue tokens.

13. What is the probability your token is in R?
14. What is the probability your token is in $\overline{R}$?
15. What is the probability your token is in $R \cup G$?
16. What is the probability your token is in $R \cap G$?
17. What is the probability your token is in G¢?
18. What is the probability your token is purple?
19. What is the probability your token is Blue or Green?
20. What is the probability your token is Blue and Green?

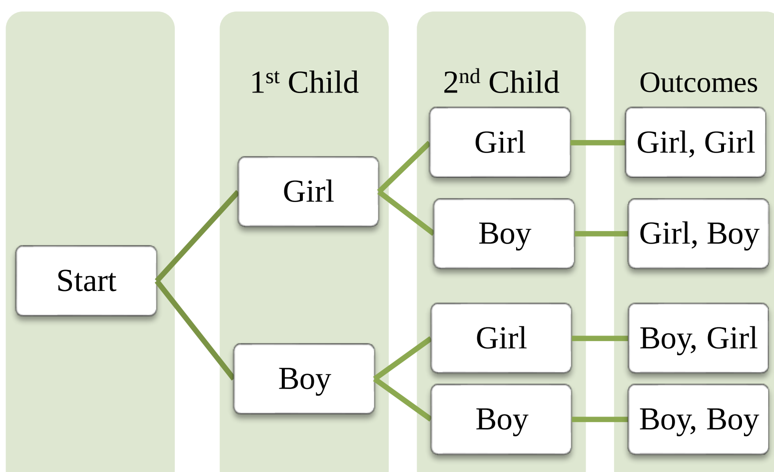
5.6 Trees and Probability

We often use tree diagrams to help us organize our sample spaces when an experiment consists of more than one thing. Tree diagrams are really just flow charts that show us all the possible outcomes for our experiment. For example say Linda is packing for a trip. She plans to take 2 pairs of pants (jeans & tan pants) and 3 shirts (red, white, & black). Every morning on her trip she will reach into her suit case and pull out something to wear.



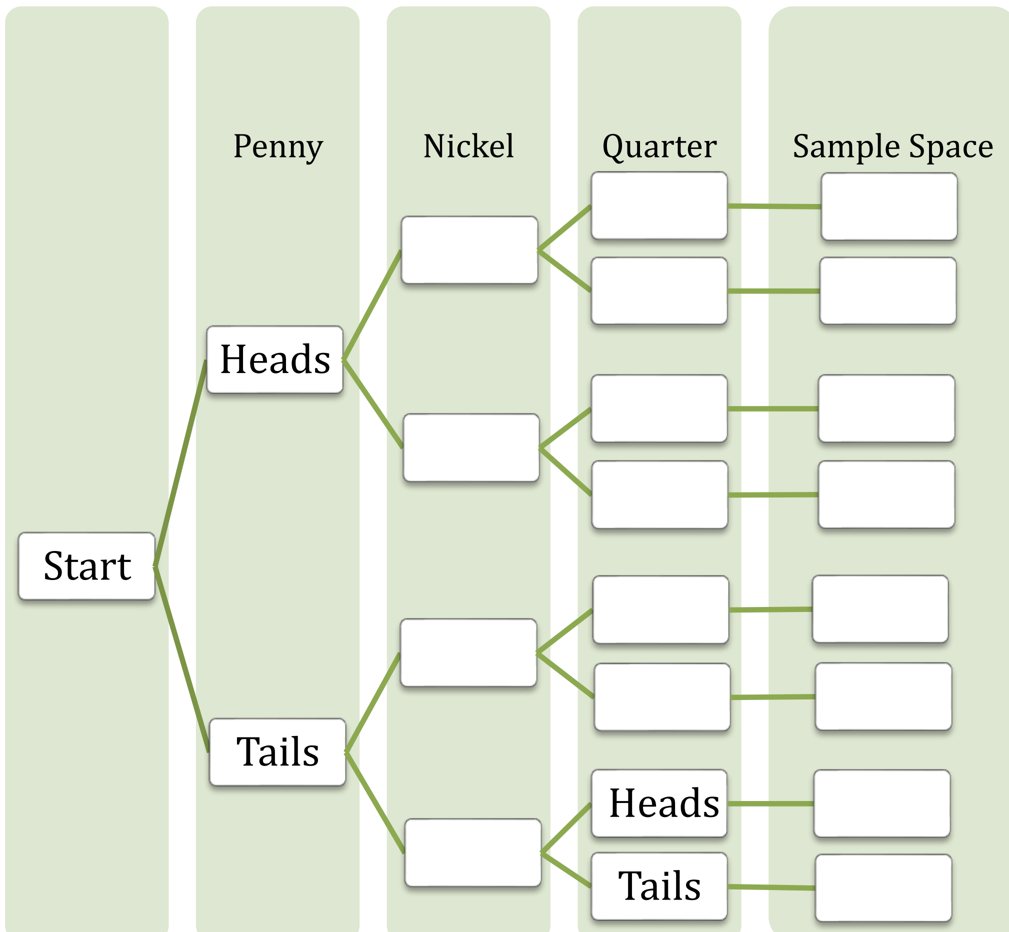
- What is the probability she is wearing tan pants and a black shirt? The answer is $\frac{1}{6}$ because out of the six outfits, she can make only one of them that matches our description.
- What is the probability she is wearing the red shirt? It is $\frac{2}{6} = \frac{1}{3}$

Here is another example of a tree that represents the birth of two children.



Classwork 5.6

Finish filling in this tree diagram to show all the outcomes in an experiment when we flip three coins: a penny, a nickel, and a quarter.



1. How many outcomes are possible? List all the outcomes.
2. What is the probability you get three heads?
3. What is the probability you get two heads and one tail in this order: HTH?
4. What is the probability you get two heads and one tail in any order?
5. What is the probability you get two heads?
6. What is the probability you get at least two heads?
7. What is the probability you get at most two heads?

5.7 Tables and Probability

Tables that show us the number of people in each category are called Frequency Tables. We can use these tables to help us answer probability questions.

<u>Example 1</u>: Use the information in the table to answer the following questions. Give probabilities as fractions and rounded to the nearest hundredth.

a) What is the probability someone chosen at random is a Democrat?
b) What is the probability someone chosen at random is a Democrat who did vote?
c) What is the probability someone chosen at random did not vote?
d) What is the probability someone chosen at random did vote and is Republican?
e) What is the probability someone chosen at random voted or is a Republican?

	Democrat	Republican	Totals
Voted	85	45	130
Did not Vote	37	12	49
Totals	122	57	179

a) There are 122 Democrats out of 179 people so the probability we choose a Democrat is $\frac{122}{179}$ » 0.68

b) There are 85 Democrats who voted out of 179 people so the probability we choose a Democrat who voted is $\frac{85}{179}$ » 0.47

c) There are 49 people who did not vote so the probability we choose a non-voter is $\frac{49}{179}$ » 0.27

d) There are 45 Republicans who voted so the probability is $\frac{45}{179}$ » 0.25

e) We need to count anyone who voted (85 + 45) and all the Republicans (45 + 12). We do not want to count the 45 Republicans who voted twice, 85 + 45 + 12 = 142 so the probability is $\frac{142}{179}$ » 0.79

We can also use tables to help us answer probability questions when we know a little something about the person chosen at random. For example in part a) above if we knew the person chosen had voted would that change the probability they are a Democrat? Yes, it would! Now we would work only with the first column of the table. Instead of looking at all 179 people we would know that the person was one of the 130 voters. This changes our denominator to 130. And our numerator would be the 85 Democrats who did vote. Knowing someone voted, the probability that they are a Democrat is $\frac{85}{130}$ » 0.65, it reduced the probability slightly. Looking at the decimal approximation makes it easier for us to compare the two probabilities.

<u>Example 2:</u> Use the information in the table to answer the following questions. Give probabilities as fractions and rounded to the nearest hundredth.

a) Knowing someone is a Democrat, what is the probability they voted?
b) Knowing someone is a Democrat, what is the probability the did not vote?
c) Given that someone did not vote, what is the probability they are a Republican?
d) Given that someone did vote, what is the probability they are a Republican?
e) Are voters more likely to be Republican than non-voters?

	Democrat	Republican	Totals
Voted	85	45	130
Did not Vote	37	12	49
Totals	122	57	179

a) Knowing we have a Democrat puts us in the left hand column of our table. There are 122 Democrats, and 85 of them voted so the probability is $\frac{85}{122}$ » 0.70

b) We are still in the left hand column of our table. There are 122 Democrats and 37 of them did not vote so the probability is $\frac{37}{122}$ » 0.30 Notice how our last two answers add up to 100%. This is because there are only two options for our Democrats: vote and not voting so these two groups must include all of our Democrats.

c) We are in the middle row (did not vote). There is a total of 49 people and 12 of them are Republicans so the probability is $\frac{12}{49}$ » 0.24

d) There were 130 people who voted and 45 of those were Republican: $\frac{45}{130} \approx 0.35$

e) Yes, voters are more likely to be Republican than those who did not vote, 35% vs 24%.

Tables can also represent **probability distributions**. A probability distribution is just a table that lists the different outcomes and the corresponding probabilities. As always, each probability is between 0 and 1. You'll also notice that if you add up all the probabilities, the sum is equal to one. This is because the table lists every possible situation that could occur.

# of Successes	Probability
0	0.4
1	0.3
2	0.2
3	0.1

Example 3: Consider the probability distribution on the right.

a) Calculate the probability of getting 1 or 2 successes.
b) Calculate the probability of getting less than 2 successes.

c) The probability of getting a 1 OR 2 is found by just adding the two corresponding probabilities. 0.3 + 0.2 = 0.5

d) The probability of getting less than 2 successes is the same as the probability of getting 0 OR 1 success. The probability of having 0 OR 1 successes is found by just adding the two corresponding probabilities. 0.4 + 0.3 = 0.7

Classwork 5.7

Use the table to answer the following questions. Give probabilities as un-simplified fractions and as decimals rounded to the nearest hundredth.

	Spanish	Not Spanish	Totals
Chinese	45	20	65
Not Chinese	18	90	108
Totals	63	110	173

1. What is the probability a student is taking Spanish?
2. What are the ODDS a student is taking Chinese?
3. What are the ODDS a student is taking both languages?
4. What is the probability a student is taking neither language?
5. What is the probability a student is taking at least one language?
6. What is the probability a student is in Chinese $\cup$ Spanish?
7. What is the probability a student is in Chinese $\cap$ Spanish?
8. Given that a student is taking Chinese, what is the probability they are also taking Spanish?
9. Looking at #1 and #8, does knowing someone is taking Chinese change the probability they are taking Spanish?
10. Give that a student is NOT taking Chinese, what is the probability they are taking Spanish?
11. Looking at #8 and #10, are students taking Chinese more likely to be taking Spanish than those not taking Chinese?
12. Knowing that someone is taking Spanish, what is the probability that they are taking Chinese?
13. Knowing that someone is not taking Spanish, what is the probability that they are taking Chinese?
14. Are students taking Spanish more likely to be taking Chinese, than those that are not taking Spanish?

Commission in \$	**Probability**
$0 \le x < 500$	0.15
$500 \le x < 1{,}000$	0.35
$1{,}000 \le x < 1{,}500$	0.4
$x \ge 1{,}500$	0.1

15. What is the probability a person makes \$1,500 or more in commissions?
16. What is the probability a person makes less than \$1,000?
17. What is the probability a person makes at least \$500, but less than \$1,000?
18. What is the probability a person makes less than \$1,500?
19. What is the probability a person makes at least \$500?

Exercises 5.7

Use the tables to answer the following questions. Give probabilities as un-simplified fractions and as decimals rounded to the nearest hundredth. Give percentages rounded to the nearest tenth of a percent.

	Do Believe in Ghosts	Do not Believe in Ghosts
Male	30	16
Female	61	23

1. What is the probability someone believes in ghosts?
2. Looking only at the females, what is the probability that a woman believes in ghosts?
3. Looking only at the males, what is the probability that a man believes in ghosts?
4. Are females more likely to believe in ghosts, than men? Why?
5. Given that someone believes in ghosts, what is the probability they are male?
6. Given that someone does not believe in ghosts, what is the probability they are male?
7. Are believers more likely to be male than non-believers? Explain, using probability.

	Pro-Life	Pro-Choice
Age > 45	40	34
Age ≤ 45	30	31

8. What is the probability someone is Pro-Life?
9. What is the probability someone is over 45 years old?
10. What is the probability someone is over 45 AND is Pro-Choice?
11. What is the probability someone is over 45 OR is Pro-Choice?
12. Looking only at those people who are 45 or younger, what is the probability that they are Pro-Choice?
13. Looking only those people who are over 45, what is the probability that they are Pro-Choice?
14. Are people over 45 more likely to be Pro-Choice? Explain.

15. What percentage of people are Pro-life?
16. What percentage of males are Pro-life?
17. What percentage of females are Pro-life?
18. Are males more likely to be Pro-life than women are? Explain.
19. Are people who are Pro-choice more likely to be female than people who are Pro-life? Use percentages to support your answer.

	Pro-life	Pro-choice
Female	14	23
Male	11	18

20. What percentage of males own sports cars?
21. What percentage of females own sports cars?
22. Are men more likely to own a sports car than women are?
23. Are people who own SUVs more likely to be female than people who own sports cars are? Use percentages to support your answer.

	SUV	Sports Car
Female	21	39
Male	135	84

Consider the probability distribution for a storm to turn into a hurricane of a given category. A category 5 hurricane is the strongest (wind speed of 155 mph or more) and category 1 is the weakest (wind speed of 74 to 95 mph).

Category	**Probability**
0 (dissipated)	3%
1	18%
2	42%
3	28%
4	7%
5	2%

24. What's the probability that a hurricane will turn into a category 3 storm?
25. What's the probability that a hurricane will be a category 3 or stronger storm?
26. What's the probability that a hurricane will be a category 1 or stronger storm?
27. What's the probability that a hurricane will be weaker than a category 3 storm?
28. What's the probability that a hurricane will turn into at most a category 4 storm?
29. What is the probability the storm will become a hurricane that is less than a category 4?
30. What is the probability the storm will become a hurricane that is not more than a category 2 storm?

Consider the probability distribution of number of boys when 5 babies are born. Answer the following questions according to the probability distribution. Notice that all the probabilities add up to 1.

# of boys	Probability
0	0.03125
1	0.15625
2	0.31250
3	0.31250
4	0.15625
5	0.03125

31. What is the probability that exactly four of the babies are boys?

32. What is the probability that no boys will be born?

33. What is the probability that at least 4 of the babies will be boys?

34. What is the probability that at least 2 boys will be born?

35. What is the probability that at most 4 of the babies will be boys?

36. If *x* = the number of baby boys, what is the probability that $x \le 3$?

37. If *x* = the number of baby boys, what is the probability that $x > 2$?

38. If *x* = the number of baby boys, what is the probability that $1 \le x \le 3$?

Pablo's favorite pick up line is "If I could rearrange the alphabet, I'd put U and I together". The probability distribution shows the number of ladies Pablo approaches before encountering one who reacts positively.

# of ladies	Probability
1	0.018
2	0.192
3	0.285
4	0.397
5	0.108

39. What is the probability of Pablo approaching 4 ladies?

40. What is the probability of Pablo approaching more than 4 ladies?

41. What is the probability of Pablo approaching at least 3 ladies?

42. What is the probability of Pablo approaching fewer than 3 ladies?

43. What is the probability he approaches a maximum of 3 ladies?

Give probabilities as un-simplified fractions and rounded to the nearest hundredth. Round percentages to the nearest tenth of a percent.

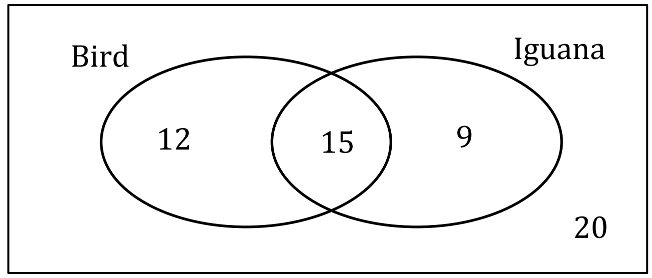
	Female	Male
Democrat	42	29
Republican	50	28

44. What percentage are female democrats?

45. Are females more likely to be Democrats than males are? Use probability to support your answer.

46. Are Republicans more likely to be males than Democrats are? Use probability to support your answer.

Use this Venn Diagram representing pet ownership of 56 people to answer the following questions. Write percentages rounded to the nearest tenth of a percent. Give probabilities as un-simplified fractions.



47. What percentage of the group owns birds?
48. What percentage of the group does not own iguanas?
49. What percentage of the group owns both animals?
50. What percentage of the group owns neither?
51. What percentage of the group is in Bird $\cap$ Iguana?
52. What percentage of the group is in Birds $\cup$ Iguana?
53. What are the ODDS that someone selected at random owns an iguana?
54. What are the ODDS that someone selected at random owns both animals?
55. What are the ODDS that someone selected at random owns a bird, but not an iguana?
56. If we know someone owns a bird, what is the probability they also own an iguana?
57. If we know someone does not own a bird, what is the probability they also own an iguana?
58. Are bird owners more likely to be iguana owners than non-bird owners? Explain using probabilities.
59. Are iguana owners more likely to be bird owners than non-iguana owners? Explain using probabilities.