Quiz intersection

Pink ① b) $P(F \,\&\, Fr) = P(F) \cdot P(Fr \mid F)$

$$= \frac{\cancel{120}^{1}}{200} \cdot \frac{50}{\cancel{120}_{1}}$$

$$= \frac{5\cancel{0}}{20\cancel{0}}$$

	Fr	S	O	Total
F	50	35	35	120

$P(F \,\&\, Fr) = .25$

Pink 1 e) are S and M independent

Show whether

$$P(S) = P(S \mid M)$$

OR $P(M) = P(M \mid S)$

$P(S) = .30$

$P(S \mid M) = .3125$

NOT THE SAME

$\therefore$ not independent

Worksheet

(1)

x	$P(x)$	$x\,P(x)$
2,000,000	0.3	600,000
750,000	0.2	150000
-500,000	0.5	-250000

$$\mu = \sum x\,P(x) = 500{,}000$$

The expected return is \$500,000.

(2)

x	$P(x)$	$xP(x)$
-3.00	0.3	$-.90$
-1.00	0.4	$-.4$
2.00	0.1	$.2$
6.00	0.2	1.2

$$\mu = \sum xP(x) = .10$$

(3)

x	$P(x)$	$x P(x)$	$x^2 P(x)$
-2	0.10	-0.20	0.40
0	0.25	0	0
1	0.30	0.30	0.30
3	0.20	0.60	1.80
4	0.15	0.60	2.40

$\sum x P(x) = 1.3$ $\quad \sum x^2 P(x) = 4.9$ $\quad \mu = 1.3$

$\mu^2 = 1.69$ $\quad \sigma^2 = 3.21$

$\sigma = 1.79$

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$

$$= 4.90 - 1.69$$

$$= 3.21$$

new for Discrete Random Variables

$$\sigma = \sqrt{3.21}$$

$$\approx 1.79$$

5.5 Factorials, Combinations and Permutations

$5!$

"5 factorial"

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

In General

$n! = n(n-1)(n-2)(n-3) \cdots 3 \cdot 2 \cdot 1$

ex Evaluate $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 40{,}320$

Notice: $1! = 1$

$0! = 1$

Combinations

Select x items from the total of n items

written ${}_nC_x$ $\binom{n}{x}$

"n choose x

(Normally written ${}_nC_r$)

(ex) Suppose I want a double scoop cone with 2 different flavors.
The ice cream parlor has 6 flavors.
How many different possibilities are there for 2 scoops?

The flavors 1 2 3 4 5 6

We could have

$\{1,2\}$, $\{1,3\}$, $\{1,4\}$, $\{1,5\}$, $\{1,6\}$, $\{2,3\}$, $\{2,4\}$, $\{2,5\}$, $\{2,6\}$

$\{3,4\}$, $\{3,5\}$, $\{3,6\}$, $\{4,5\}$, $\{4,6\}$, $\{5,6\}$

15 possibilities

Combinations

$${}_nC_x = \frac{n!}{x!\,\underline{(n-x)!}}$$

$${}_6C_2 = \frac{6!}{2!\,(6-2)!}$$

$$= \frac{\cancel{6}^{3} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{2} \cdot 1\,(\cancel{4 \cdot 3 \cdot 2 \cdot 1})}$$

$$= 15$$

$$= \frac{6!}{2!\,(4!)} = \frac{\cancel{6}^{3} \cdot 5}{\cancel{2}_{1} \cdot 1} = 15$$

Short method

(ex) Our math department used to have 18 full time instructors. We need to have 3 of them serve on a hiring committee for a new dean.

How many possibilities are there for choosing 3 people?

Solution

$n = 18$

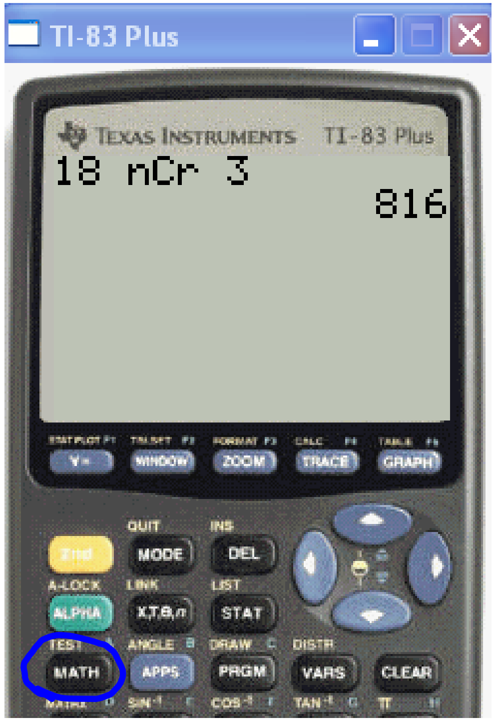
$x = 3$

$$ {}_{18}C_3 = \frac{18!}{3!(15!)} = \frac{(18\cdot 17\cdot 16)\cancel{15!}}{(3\cdot 2\cdot 1)\cancel{15!}} $$

$$ = \frac{\overset{3}{\cancel{18}}\cdot 17\cdot 16}{\cancel{6}_1} $$

$= 816$ possible combinations

Look at Calculator.



Some things are obvious

We have 6 people. How many ways can we choose 6?

$${}_6C_6 = \frac{\cancel{6!}^{1}}{\cancel{6!}_{1}(0)!} = 1$$

How many ways can we choose zero?

$${}_6C_0 = \frac{6!}{0!\,6!} = 1$$

With combinations order does not matter

When order does matter we use Permutations

(ex) Suppose we want to choose, president, vice president & secretary for our club

Club of 6 people

1st person - pres
2nd - vp
3rd - sec

1 2 3 4 5 6

permutations

1 2 3 — 1
1 3 2 — 2
2 1 3 — 3
2 3 1 — 4
3 1 2 — 5
3 2 1 — 6

same 3 people

6

Choose 3 people

$$ {}_6C_3 = \frac{6!}{3!(3!)} = \frac{\cancel{6}\cdot 5\cdot 4}{\cancel{3\cdot 2}\cdot 1} $$

$$ = \frac{\cancel{6}\cdot 5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{(\cancel{3\cdot 2}\cdot 1)(\cancel{3\cdot 2\cdot 1})} $$

= 20 combinations

= 20 (6)

20 (3!)

$$ {}_6C_3 \cdot 3! = {}_6P_3 $$

$$ \frac{n!}{\cancel{x!}(n-x)!} \cdot \frac{\cancel{x!}}{1} $$

$$ {}_nP_x = \frac{n!}{(n-x)!} $$

$$ {}_6P_3 = \frac{6!}{3!} $$

$$ = \frac{6\cdot 5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{\cancel{3\cdot 2\cdot 1}} $$

= 120

$3! = 3\cdot 2\cdot 1$

$3!(3!)$

$(3\cdot 2\cdot 1)(3\cdot 2\cdot 1)$

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