

unbiased estimator

$$\mu_{\bar{x}} = \mu$$

consistent estimator

$\sigma_{\bar{x}}$ decreases as n increases

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{true if}$$

$$\frac{n}{N} \leq .05$$

7.4

\bar{x} the mean of a sample

The shape of the Sampling Distribution
of \bar{x}

Two cases

① The population from which the sample is taken is normally distributed.

$$\text{with } \mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

assuming
 $\frac{n}{N} \leq .05$

then the distribution of \bar{x}
is normally distributed
regardless of the sample
size.

#7.32
pg. 311

$$\mu = 6.7 \text{ min}$$

$$\sigma = 2.1 \text{ min}$$

$$n = 16$$

$$\mu_{\bar{x}} = 6.7$$

$$\sigma_{\bar{x}} = \frac{2.1}{\sqrt{16}} \approx 0.525$$

It is normally distributed

② The population is not normally distributed

Central Limit Theorem -

For a large sample size, the sampling distribution of \bar{x} is approximately normal, irrespective of the shape of the population distribution.

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{if } \frac{n}{N} \leq .05$$

and the sample size is considered large if $n \geq 30$ *

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7.36

$\mu = 133$ lbs $\sigma = 24$ lbs
skewed right

$n = 45$ persons

$$\mu_{\bar{x}} = 133 \quad \sigma_{\bar{x}} = \frac{24}{\sqrt{45}} \approx 3.578$$

The shape is approximately
normal.

7.5 Applications

pg 312

$$\begin{aligned} P(\mu - \sigma_{\bar{x}} \leq \mu \leq \mu + \sigma_{\bar{x}}) &\approx \\ &= \text{normalcdf}(-1, 1, 0, 1) \\ &\approx .682689 \\ &\approx .6827 \end{aligned}$$

EX 7-5 pg 313

weights of packages of cookies
(normally distributed)

$$\mu = 32 \text{ oz.} \quad \sigma = .3 \text{ oz}$$

Find $P(31.8 < \mu_{\bar{x}} < 31.9)$ if $n = 20$

$$\mu_{\bar{x}} = 32 \text{ oz}$$

$$\frac{.3}{\sqrt{20}}$$

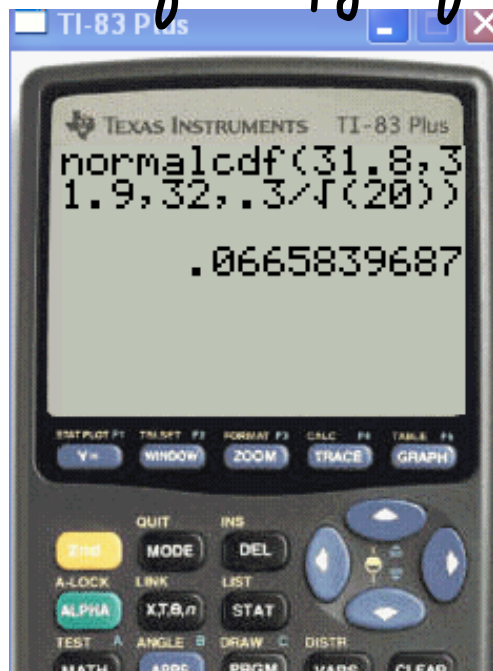
$$\sigma_{\bar{x}} = \frac{.3}{\sqrt{20}}$$

$$P(31.8 < \mu_{\bar{x}} < 31.9) = \text{normalcdf}(31.8, 31.9, 32, \underline{\underline{.3/\sqrt{20}}})$$

$$\approx .0665839$$

$$\approx .0666$$

from pg before



7-6

Avg of \$7868 on credit
in cards $\sigma = 2160$
 $n = 81$

Shape of the distribution
is unknown

a) within \$440 of pop mean.

$$7868 - 440 = 7428$$

$$7868 + 440 = 8308$$

$$P(7428 < \mu_{\bar{x}} < 8308)$$

$$= \text{normalcdf}(7428, 8308, 7868, 2160/\sqrt{81})$$

$$\approx .9332$$

Shape - approximately
normal.

$$\begin{aligned} \text{b) } P(-\infty < \bar{x} < 7548) & \quad 2868 - 320 \\ & \quad = 7548 \\ & = \text{normalcdf}(-E99, 7548, 2868, 2160/\sqrt{81}) \\ & \approx .0912 \end{aligned}$$

\bar{x}
one

7.6

Population and Sample Proportions.

$$\text{Pop proportion} = \frac{\# \text{ with the desired characteristics}}{\text{total \# in population}}$$

$$p = \frac{X}{N}$$

$$\text{the sample proportion} = \frac{\# \text{ with trait}}{\text{total in sample}}$$

$$\hat{p} = \frac{x}{n}$$

"p hat"

#7.60 population of 1000, 640 possess
a certain characteristic

In a sample of 40, 24 possess
that characteristic

Find p and \hat{p}

$$p = \frac{X}{N} = \frac{640}{1000} = .64$$

$$\hat{p} = \frac{24}{40} = .60$$

The sampling error is

$$\hat{p} - p$$
$$.60 - .64 = -.04$$

The mean of the sample proportions, \hat{p}
is equal to the population
proportion, p .

unbiased estimator

$$\mu_{\hat{p}} = p$$

The standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

$$q = 1 - p$$

where p is the population proportion!

$$\frac{n}{N} \leq .05$$

$$\sqrt{\frac{N-n}{N-1}}$$

consistent estimators

$\sigma_{\hat{p}}$ gets smaller as n gets

Central Limit Theorem (larger.
for Proportions)

The sample distribution of \hat{p} is
approximately normal for sufficiently
large sample size

$$np > 5 \quad \text{and} \quad nq > 5$$

EX 7-9

50% of Americans are satisfied
their jobs

(assume this is true for
the current population
of Americans)

Let \hat{p} be the proportion of
Americans in a r.s. of 1000 who
are satisfied.

Find the mean, standard deviation
and describe the shape

$$\mu_{\hat{p}} = .5$$

$$n = 1000$$

$$np = 1000(.5) = 500$$

$$nq = 500$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(.5)(.5)}{1000}} \approx .0158$$

shape is
approx
normal

What if $p = .65$ $q = .35$