KEY

# Confidence Intervals and Hypothesis Tests for $\mu_1 - \mu_2$ (§§ 10.1, 11.1)

If independent SRS's are drawn from normally distributed populations, a confidence interval estimate for the difference in the population means, $\mu_1 - \mu_2$, and the test statistic for a hypothesis test about the difference in the population means $\mu_1 - \mu_2$, ($H_0$: $\mu_1 - \mu_2 = \Delta_0$, the TI calculators use only the most common case: $\Delta_0 = 0$) are:

1) not assuming that the population variances are equal, ("unpooled" *t*-procedures)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} SE(\bar{x}_1 - \bar{x}_2) \text{ and } t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{SE(\bar{x}_1 - \bar{x}_2)}, \quad SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

(use Welsh's formula or df provided by a calculator or software).

On TI calculators CI with `0:2-SampTInt…` and `Pooled:No`
On TI calculators test for $\Delta_0 = 0$ with `4:2-SampTTest…` and `Pooled:No`

2) If we assume that the population variances are equal, ("pooled" *t*-procedures)
(We won't use this for reasons stated in the book in the last paragraph on page 493)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} SE_{pooled}(\bar{x}_1 - \bar{x}_2) \text{ and } t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{SE_{pooled}(\bar{x}_1 - \bar{x}_2)} \text{ where } df = n_1 + n_2 - 2 \text{ and}$$

$$SE_{pooled} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \text{ for } s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Note that the degrees of freedom for these "pooled" procedures are at least as large as they are for the "unpooled" procedures.

On TI calculators CI with `0:2-SampTInt…` and `Pooled:Yes`
On TI calculators test for $\Delta_0 = 0$ with `4:2-SampTTest…` and `Pooled:Yes`

**The producer of Gobbler frozen turkey dinners claims that their new "reduced fat" dinner contains less fat than Big Bird frozen turkey dinners. An independent research group takes a sample of 12 Gobbler dinners and 15 Big Bird dinners and analyzes the fat content (in grams). The results are summarized below:**

| | Gobbler | Big Bird |
|---|---|---|
| $n$ | 12 | 15 |
| $\bar{x}$ | 7.4 g | 9.6 g |
| $s$ | 1.6 g | 2.7 g |

A. Use your TI calculator (`0:2-SampTInt…`) to obtain a 98% confidence interval for $\mu_{BB} - \mu_G$, the difference in the population means.

Give the point estimate of $\mu_{BB} - \mu_G$ 9.6−7.4 2.2

pooled = No (df = 23.3 ) (.11132, 4.2887)

pooled = Yes (df = 25 ) (.00336, 4.3966)

B. Do the data provide statistically significant evidence that the producer of Gobbler dinners claim is true?

1. State appropriate null and alternative hypotheses.

$(\mu_{BB}-\mu_G=0)$ $(\mu_{BB}-\mu_G)>0$

$H_0$: $\mu_{BB}=\mu_G$ $H_A$: $\mu_{BB}>\mu_G$

2. Using the "unpooled" procedure on your calculator,

a. the degrees of freedom are $df=$ 23.3

b. the observed value of $t$ is 2.631

c. the $P$-value is .0074

d. Write a properly worded and meaningful conclusion.

The is sufficient evidence (strong, P=.007) that the mean fat content of the Big Bird dinners is higher than that for the Gobbler dinners

3. Using the "pooled" procedure on your calculator,

a. the degrees of freedom are $df=$ 25

b. the observed value of $t$ is 2.489

c. the $P$-value is .0099

d. is the conclusion different from above? If so, write a properly worded and meaningful conclusion.

Same as above

KEY

## Inferences for a difference in proportions (§§ 10.2 and 11.2)

If two independent (sufficiently large) SRS's are drawn from two populations, sizes $n_1$ and $n_2$, a confidence interval estimate for the difference in the population proportions, $p_1 - p_2$, is given by

$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}(SE)$ where $SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ and if we are testing the null hypothesis $p_1 = p_2$ then the test statistic is a $z$ statistic given by $z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{SE_{pooled}}$, with $SE_{pooled} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ is the pooled estimate of the common population proportion, where $x_1$, and $x_2$ are the number of success in the samples of sizes $n_1$ and $n_2$ from populations 1 and 2 respectively.

Note that in practice technology (calculator or computer software) will be used to get these values.

**I. A psychologist believes that since artistic endeavors are a right brain activity a higher proportion of people in the arts would be left-handed. To test this theory she takes a sample of 200 artists (painters, writers, musicians, etc.) and 300 non-artists. She finds that among the artists 26 are left-handed and among the non-artists 24 are left handed? (I'll demonstrate doing it by hand, you will use the TI calculator or Minitab.)**

A. Construct a 90% confidence interval estimate for the difference in the proportion of artists who are left-handed and the proportion of non-artists who are left-handed. (`B:2-PropZInt`)

`B:2-PropZInt...x1:` 26 `, n1:` 200 `, x2:` 24 `, n2:` 300 `, C-Level:` .9 non artists

artists — $\hat{p}_A$ = .13 , $\hat{p}_B$ = .08

CI (Don't round): (.00316, .09684)

B. Complete the following hypothesis test of the psychologist's claim. (`6:2-PropZTest`)

1. State the assumptions. SRS (independent)
sufficient large sample sizes

2. State appropriate null and alternative hypotheses.

$H_0$: $p_A = p_B$ $H_A$: $p_A > p_B$

3. Compute the value of the appropriate test statistic. (Use your calculator).

$\frac{26+24}{200+300} = \frac{50}{500} = .1$

What is the value of the pooled estimate of $p$, $\hat{p}$? .1

4. Give the $P$-value. .034

5. What conclusion would you draw?

The is sufficient evidence to conclude that there is a higher proportion of artists than non-artists who are left handed.

# Goodness-of-Fit

The distribution of grades at a distinguished university are given in the table below:

| Grade | Percentage |
|---|---|
| A | 18% |
| B | 24% |
| C | 36% |
| D | 14% |
| F | 8% |

The administration claims that the grade distribution at Solano College is about the same as at this distinguished university. A sample of grades from Solano College find the following grades.

| Grade | Count |
|---|---|
| A | 214 |
| B | 231 |
| C | 267 |
| D | 112 |
| F | 76 |

900

Use a goodness-of-fit test to determine if this sample came from a population with the same grade distribution given above for the distinguished university.

| Grade | Expected (E) | Observed (O) | O – E | $(O - E)^2/E$ |
|---|---|---|---|---|
| A | .18×900=162 | 214 | 52 | ≈16.7 |
| B | .24×900=216 | 231 | 15 | ≈1.0 |
| C | .36×900=324 | 267 | −57 | ≈10.0 |
| D | .14×900=126 | 112 | −14 | ≈1.6 |
| F | .08×900=72 | 76 | 4 | ≈9.2 |

0

1. Compute $\chi^2$. 38.5

2. $H_0$: The grade dist. is as stated in the 1st table

   $H_A$: " " " is not " " " " "

3. How many degrees of freedom are there for the Chi-square test? 4

4. Compute the $P$-value. $\chi^2$cdf(38.5, 10^99, 4) $P$-value = $8.8 \times 10^{-8}$

5. Write a properly worded and meaningful conclusion.

   There is very strong evidence the grade dist. at SCC is not the same as it is at the distinguished university.

# A Chi-square Test For Independence

The records of a sample of SCC statistics students are recalled and they are cross-classified by their preparation for the class (the highest prerequisite course they completed) and their grade for the statistics course. The results of the study are in the following two-way table.

1. Fill in the expected values for each cell (use the Minitab printout below for most cells).

| | Statistics Grade | | | | |
|---|---|---|---|---|---|
| Prereq. Course | A | B | C | Other | from Minitab below |
| Algebra II | 78 (100) | 115 (150) | 206 (150) | 101 (100) | 500 |
| College Algebra | 62 (80.0) | 164 (120) | 88 (120) | 86 (80) | 400 |
| Calculus | 60 (20) | 21 (30) | 6 (30) | 13 (20) | 100 |
| | 200 | 300 | 300 | 200 | 1000 |

2. Verify that (observed – expected)$^2$/expected for the (1, 1) cell is 4.840. This is the contribution of this cell to the Chi-square statistic. Minitab analysis produces:

## Rows: Prereq Columns: Statistics Grade

| | A | B | C | Others | All |
|---|---|---|---|---|---|
| Alg II | 78 | 115 | 206 | 101 | 500 |
| | 100 | 150 | 150 | 100 | |
| | 4.840 | 8.167 | 20.907 | 0.010 | |
| Col Alg | 62 | 164 | 88 | 86 | 400 |
| | 80 | 120 | 120 | 80 | |
| | 4.050 | 16.133 | 8.533 | 0.450 | |
| Calc | 60 | 21 | 6 | 13 | 100 |
| | 20 | 30 | 30 | 20 | |
| | 80.000 | 2.700 | 19.200 | 2.450 | |
| All | 200 | 300 | 300 | 200 | 1000 |

*Cell Contents*
*Count*
*Expected count*
*Contribution to Chi-square*

## Chi-Square Test

| | Chi-Square | DF | P-Value |
|---|---|---|---|
| Pearson | $\boxed{167.440}$ | 6 | 0.000 |
| Likelihood Ratio | 147.885 | 6 | 0.000 |

3. State an appropriate null and alternative hypotheses.

$H_0$: Prerequicite course and statistics grade are independent.

$H_A$: Prerequisite course and statistics grade are not independent.

4. What is the observed value of the test statistic? 167.440

5. How many degrees of freedom are there? 6

6. Compute the P-value? 0.000

7. Write a complete, carefully worded conclusion?

There is very strong evidence that the prerequicite course and their statistics grade are not independent.

# A Chi-Square Test For Homogeneity

**A sociologist is interested in the relationship between marital status and different groups of workers. She surveys 200 blue collar workers, 300 white-collar workers, and 150 college professors, partial Minitab 14 results are given below.**

| | Blue-collar | White-collar | Professional | Total |
|---|---|---|---|---|
| Married | 103 | 144 | 88 | 335 |
| Single | 42 | 65 | 17 | 124 |
| Divorced | 45 | $\boxed{86}$ | 37 | 168 |
| Widowed | 10 | 5 | 8 | 23 |
| Total | 200 | 300 | 150 | 650 |

```
Chi-Sq =  0.000 +  0.729 +  1.479 +
          0.388 +  1.055 +  4.715 +
          0.866 +  0.923 +  0.081 +
          1.207 +  2.970 +  1.366 = 15.779
```
$= \chi^2$

1. State appropriate null and alternative hypotheses for this test.

   $H_0$: The distribution of marital status is the same for all work categories.

   $H_A$: The distribution of marital status is not the same for all work categories

2. What is the expected count for the third row, second column cell (divorced white collar workers)? (Show work) $\frac{\text{row total} \times \text{column total}}{\text{grand total}} = \frac{168 \times 300}{650}$

   77.5

3. What is the computed value of $\chi^2$. $\chi^2 =$ 15.779

4. How many degrees of freedom are there for this test? $(4-1)\times(3-1)$ $3 \times 2$ $Df =$ 6

5. Use your calculator's $\chi^2$cdf to compute the $P$-value.

   input: $\chi^2$cdf ( 15.779, 10^99, 6 ) )

   $P$-value .015

6. The initial or formal conclusion (Reject $H_0$/Fail to reject $H_0$) Reject $H_0$

7. Write a properly worded and meaningful conclusion. (Use complete sentences please and recall that this must be an answer to the question at hand and not just about null or alternative hypotheses. Do not use the words "null", "alternative", or "hypothesis" in your conclusion.)

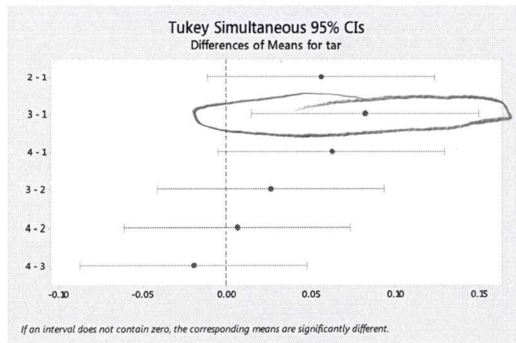
   There is sufficient evidence to conclude that marital status varies by work category.

# ANOVA

**Four brands of cigars were tested for tar content. Minitab was used to perform a One-Way ANOVA on the experimental results to determine if there was a difference among the mean tar content of the different brands. The results are below:**

Analysis of Variance

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|---|---|---|---|---|---|
| brand | 3 | 0.09260 | 0.030867 | 3.72 | 0.014 |
| Error | 96 | 0.79670 | 0.008299 | | |
| Total | 99 | 0.88930 | | | |




1. State appropriate null and alternative hypotheses.

$H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A$: not all means are equal (not $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$)
i.e. at least one pair of means are not equal

2. What are the degrees of freedom for this test? (numerator *df*, denominator *df*) ( 3 , 96 )

3. Use your calculator's Fcdf function to find the *P*-value for this test?

input: Fcdf( 3.72, 10^99, 3, 96 ) )

*P*-value = .014021...

4. Write a properly worded and meaningful conclusion. (Use complete sentences please and recall that this must be an answer to the question at hand and not just about null or alternative hypotheses. Do not use the words "null", "alternative", or "hypothesis" in your conclusion.)

There is sufficient evidence ($P = .014$) that not all of the mean tar contents are equal.

5. If you concluded that not all the means are equal, based on the 95% confidence intervals in the above printout which pair(s) of cigar (if any) seem to have different mean tar content?

$\mu_1 \neq \mu_3$