Mean and Variance of Discrete Random Variables

Let X be a discrete random variable. For convenience and brevity we will write the $P(X = x)$ as $P(x)$.

Definition: The ***expected value*** of a discrete random variable or the ***mean*** of its distribution is defined as $E(X) = \mu = \Sigma xP(x)$. Note the use of the Greek letter indicating that this is a parameter, not a statistic, it is the mean of a theoretical distribution (the probability model), not the mean of sample data.

Definition: The ***variance*** of a discrete probability distribution is defined by $Var(X) = \sigma^2 = \Sigma(x - \mu)^2 P(x)$.

A computation formula is $\sigma^2 = \Sigma x^2P(x) - \mu^2$. The standard deviation is $SD(X) = \sqrt{Var(X)} = \sigma = \sqrt{\sigma^2}$.

Exercises:

1. A company is considering taking on a project that has a 0.3 probability of producing a profit \$2,000,000, a probability of 0.2 of producing a profit of \$750,000, and a probability of 0.5 of creating a loss of \$500,000. What is the company's expected return on this project?

X	P(X)
2,000,000	.3
750000	.2
−500,000	.5

$E(X) = 2000000(.3) + 750000(.2) - 500000(.5)$

$= 600{,}000 + 150\,000 - 250000$

$= \$500\,000$

2. A game is played in which a you can lose \$3 with a probability of 0.3, lose \$1 with a probability of 0.4, win \$2 with a probability of 0.1 and win \$6 with a probability of 0.2. Make a table for the distribution of the random variable X = the amount you win (a loss is a negative win, *e.g.* lose \$1 = −1). Compute the expected value of X. Would you be willing to play this game? Explain why.

X	P(X)	X P(X)
−3	.3	−.9
−1	.4	−0.4
2	.1	.2
6	.2	1.2
		0.1

You'd "expect" to win \$0.10 per play. Of course you'd play as often as possible, you'd expect to gain \$.10 per play

3. Complete the following table and find the mean, variance, and standard deviation of the probability distribution.

	x	$P(x)$	$xP(x)$	$x^2P(x)$
	−2	0.10	−.2	.4
	0	0.25	0	0
	1	0.30	.3	.3
	3	0.20	.6	1.8
	4	0.15	.6	2.4
Σ			1.3	4.9

$\mu =$ 1.3

$\sigma^2 =$ 3.21

$\sigma =$ 1.792

μ

$\sigma^2 = 4.9 - 1.3^2$

$4.9 - 1.69 = 3.21$