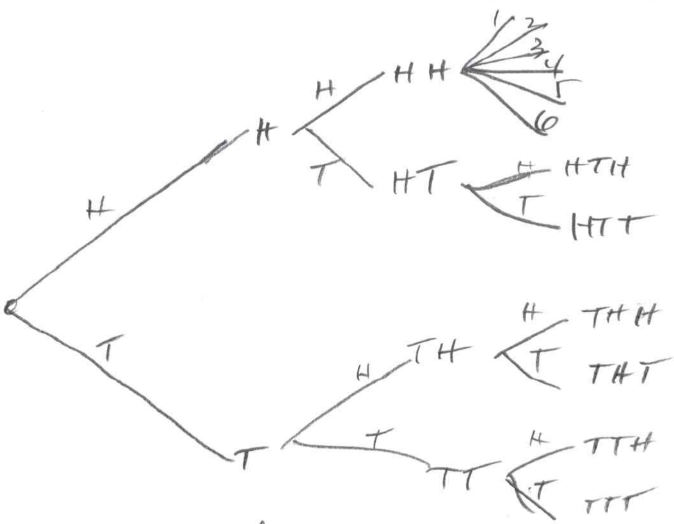
KEY

Probability 1

I. An experiment consists of two steps: first flipping two coins and then if the coins both land heads up a die is rolled otherwise a coin flipped. (Outcomes are listed like HH3 or THH.)

1. Make a tree diagram and write out (using proper set notation) the sample space for this experiment.



S = { HH1, HH2, HH3, HH4, HH5, HH6, HTH, HTT, THH, THT, TTH TTT }

2. Write out (using proper set notation) the event that exactly two tails appear.

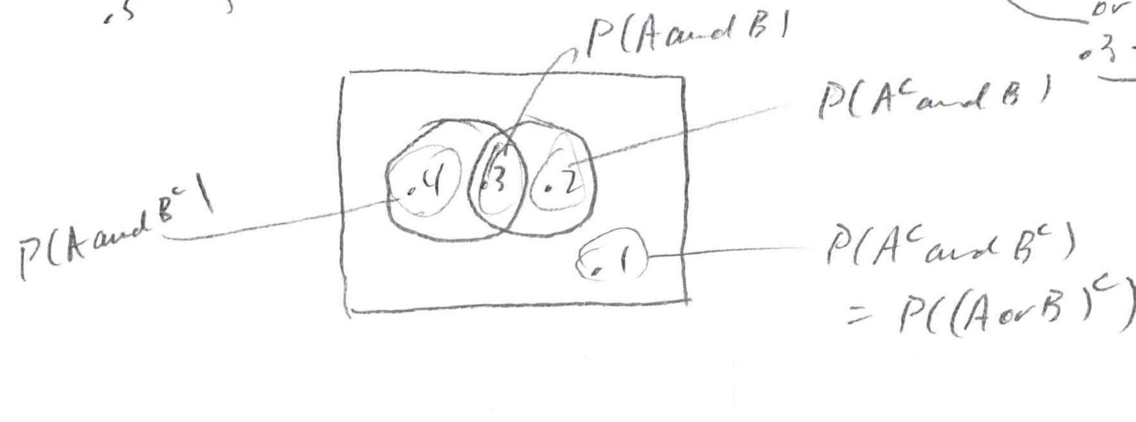
{ HTT, THT, TTH }

3. Write out (using proper set notation) the event that an odd number is rolled on the die.

{ HH1, HH3 , HH5 }

II. Let $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \text{ and } B) = 0.3$, use the rules of probability to find each of the following:

1. $P(A^c)$ $1 - .7$ $.3$
2. $P(B^c)$ $1 - .5$ $.5$
3. $P(A \text{ or } B)$ $.7 + .5 - .3$ $.9$
4. $P(A \mid B)$ $\frac{.3}{.5} = \frac{3}{5} =$ $.6$
5. $P(B \mid A)$. $\frac{.3}{.7} = \frac{3}{7} \approx$ $.43$
6. $P(A^c \text{ and } B)$ $.5 - .3$ $.2$
7. $P(A^c \text{ or } B)$ $.2 + .1 + .3$ $.6$ or $.3 + .5 - .2$
8. $P((A \text{ or } B)^c)$ $1 - .9$ $.1$



KEY

III. **A recent poll of voters on the ERA (Equal Rights Amendment) produced the results in the table below:**

	For	Against	
Male	100	100	200
Female	210	90	300
	310	190	500

Assuming that one of these voters is selected at random find:

1. P(male) $\frac{200}{500} = \frac{2}{5}$ $.40$

2. P(male for the ERA) $\frac{100}{500} = \frac{1}{5}$ $.20$

3. P(male | for the ERA) $\frac{100}{310} = \frac{10}{31} \approx$ $.32$

4. P(for the ERA | male) $\frac{100}{200} = \frac{1}{2}$ $.50$

5. Use the above probabilities to show that gender and being for the ERA are not independent.

#3 $\neq$ #1

P(male | for) $\neq$ P(male) $\therefore$ not indep.

IV. **A box contains 5 red and 3 white balls.**

1. If two balls are drawn at random *without* replacement (i.e. a first ball is draw, not returned to the box and then a second ball is drawn), find the probability:

   a) the first is red and the second is white $P(RW) = P(R)P(W|R) = \frac{5}{8}\cdot\frac{3}{7}$ $\frac{15}{56} \approx .27$

   b) they are both red $P(RR) = P(R)P(R|R) = \frac{5}{8}\cdot\frac{4}{7}$ $\frac{20}{56} = \frac{5}{14} \approx .36$

   c) they are both white $P(WW) = P(W)P(W|W) = \frac{3}{8}\cdot\frac{2}{7} = \frac{3}{28}$ $3/28 \approx .11$

   d) there is one of each color drawn $P(RW)+P(WR) = \frac{15}{56}+\frac{15}{56} = \frac{30}{56}$ $15/28 \approx .54$

2. If two balls are drawn at random *with* replacement (i.e. a first ball is draw, it is returned to the box and then a ball is again drawn), find the probability:

   a) the first is red and the second is white $\frac{5}{8}\cdot\frac{3}{8}$ $\frac{15}{64} \approx .23$

   b) they are both red $\frac{5}{8}\cdot\frac{5}{8}$ $25/64 \approx .39$

   c) they are both white $\frac{3}{8}\cdot\frac{3}{8}$ $9/64 \approx .14$

   d) there is one of each color drawn $2 \times \frac{15}{64}$ $15/32 \approx .47$

V. **Given that $P(A) = 0.2$ and $P(B) = 0.7$**

1. find $P(A$ and $B)$ given that

   a) A and B are mutually exclusive no overlap 0

   b) A and B are independent $P(A) \times P(B) = .2 \times .7$ $.14$

2. find $P(A$ or $B)$ given that

   a) A and B are mutually exclusive $.2 + .7$ $.90$

   b) A and B are independent $.2 + .7 - .14$ $.76$